XX. Indication of Cold Dark Matter: Rotational Velocity Curves - Milky Way Galaxy

Observed Rotational Velocity of Galaxies - Velocity Does Not Falloff Rapidly Like Planets

Observing the rotational velocity of stars in galaxies is a fundamental tool to derive the mass distribution in the galaxy. Estimating the velocity of galaxy based on visible based on Classic Newton's or Kepler's Law's gives a velocity curve (VR_{Kep}) that falls off quickly with distance. The actual Galactic Velocity acts like there is a halo of matter around galaxy.

Cold Dark Matter constitutes about 26.5% of the mass–energy density of the universe. The remaining 4.9% comprises all ordinary matter observed as atoms, chemical elements, gas and plasma, the stuff of which visible planets, stars and galaxies are made. The great majority of ordinary matter in the universe is unseen, since visible stars and gas inside galaxies and clusters account for less than 10% of the ordinary matter contribution to the mass–energy density of the universe.

We want to calculate the Fraction of Cold Dark Matter in the Milky Way Galaxy

Bright Matter Mass of Milky Way Galaxy: $M_{mwg} := 6.3 \cdot 10^{41} \cdot kg \cdot 0.1$ $\quad kpc := 3.08 \cdot 10^{16} km$

Radial Scale Length: $R_0 := 2.1 kpc$ $\quad r_c := 16 kpc$ $\quad M_o := 6 \cdot 10^{42} kg$

Expected Galactic Velocity Distribution (VKep) based on Keplerian type (Sun - Planetary) Mass Distribution

This is the type of falloff of velocity with distance we would expect to see from the mass of ordinary visible matter

$VR_{Kep} := READPRN(\text{"Galaxy Expected.csv"})$ $\quad R_{Kep} := VR_{Kep}^{\langle 0 \rangle} \cdot 4$ $\quad X := 1 - 0.7 \dfrac{R_{Kep}}{100}$

$V_{Kepler} := \overrightarrow{\left(VR_{Kep}^{\langle 1 \rangle} \cdot X\right)}$

See Graph of Galaxy Velocity on Next Page

Determination of Amount of Dark Matter from Rotation Curve (RC) of Milky Way Galaxy

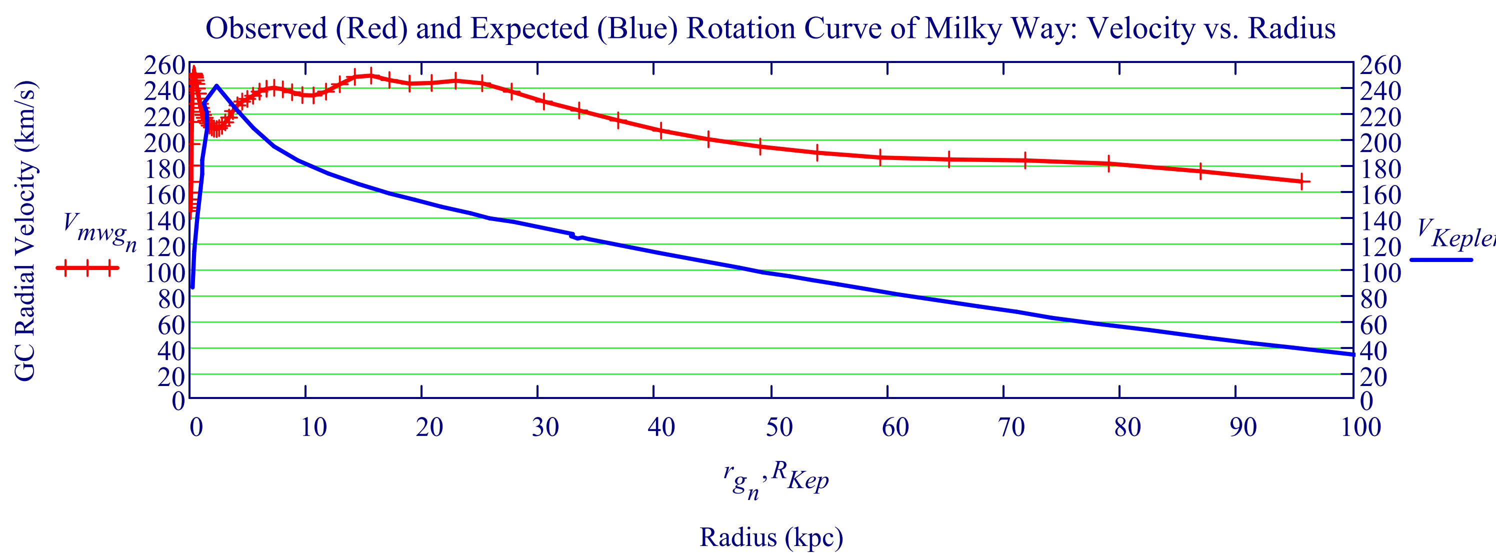
Radius (kpc), $V_{rotation}$ (kms/s), Std Dev (km/s)

DATA: Rotation Curve Parameters of the Milky Way and the Dark Matter Density, Yoshiaki Sofue, mdpi.com
Institute of Astronomy, Graduate School of Sciences, The University of Tokyo, Mitaka, Tokyo, Japan

Read Data for Rotation Curve: $RCMW := READPRN(\text{"Rotation curve of the Milky Way.txt"})$

Milky Way Velocity: $V_{mwg} := RCMW^{\langle 1 \rangle}$ $\quad$ Let r_g be the radius of Galaxy: $r_g := RCMW^{\langle 0 \rangle}$ $\quad n := 0 .. rows(RCMW) - 1$

Note the two prominent rotation velocity dips at radii 3 and 9 kpc.

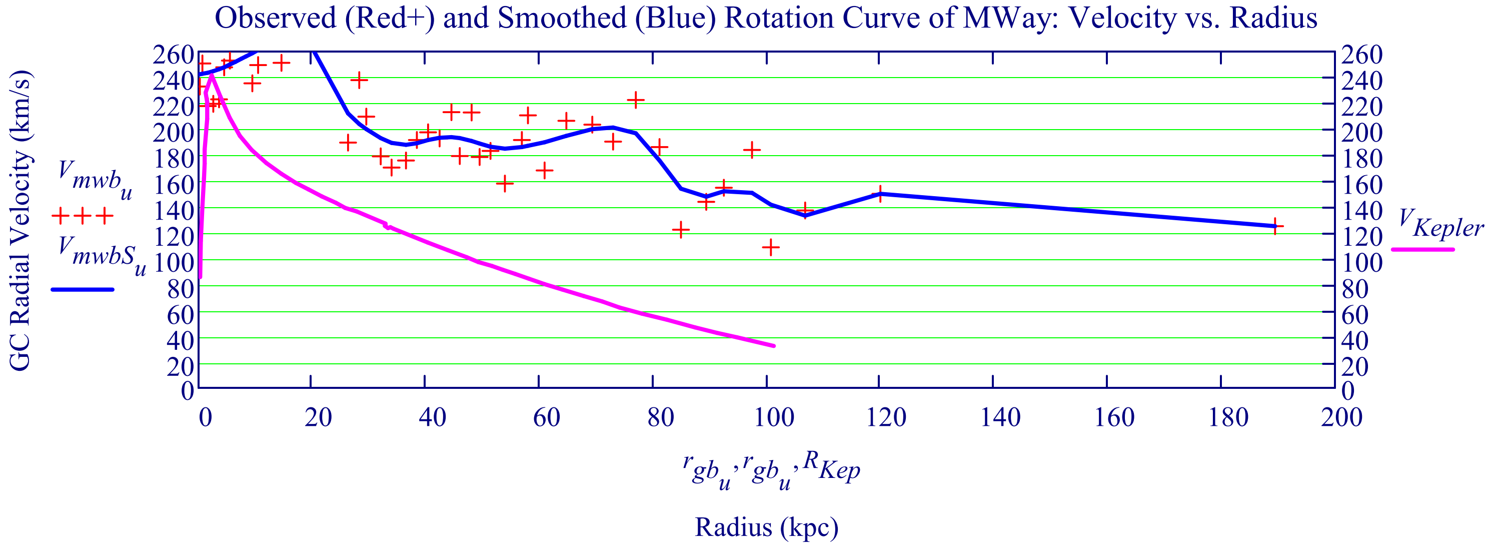


ROTATION CURVE OF THE MILKY WAY OUT TO ~200 kpc

https://iopscience.iop.org/article/10.1088/0004-637X/785/1/63/pdf

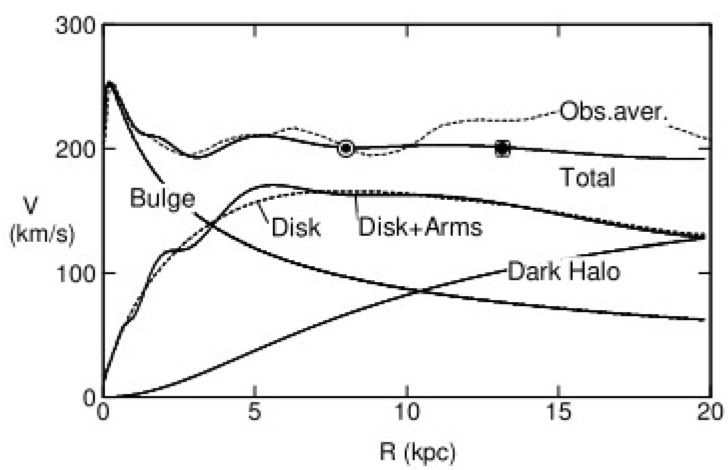
$MWB := READPRN(\text{"RC MILKY WAY 200 kpc -Bhattacharjee.txt"})$

$V_{mwb} := MWB^{\langle 1 \rangle}$ $\quad r_{gb} := MWB^{\langle 0 \rangle}$ $\quad u := 0, 1 .. rows(MWB) - 1$ $\quad V_{mwbS} := ksmooth(r_{gb}, V_{mwb}, 12)$



Composite Rotation Curve of Milky Way Galaxy Showing Mass Components

Composite Rotation Curve including the bulge, disk, spiral arms, and dark halo.
Yoshiaki Sofue, Mareki Honma , and Toshihiro Omodaka, PASJ 2018



The rotation velocity is written by the gravitational potential as $V(R) = \sqrt{R \cdot \frac{\partial}{\partial R} \Phi}$

where $\Phi = \sum_i \Phi_i$

with Φi being the potential of the i-th mass component

Knowing that Vi(R) = R ∂ Φi / ∂ R, we have . $V(R) = \sqrt{\sum_i V_i^2}$

Mass Components

Below, the subscript BH represents black hole, b stands for bulge, d for disk, and h for the dark halo. The contribution from the black hole can be neglected in sufficiently high accuracy, when the dark halo is concerned.

$$V(R) = \sqrt{V_{BH}(R)^2 + V_b(R)^2 + V_d(R)^2 + V_h(R)^2}.$$

The **mass components** are usually assumed to have the following functional forms:

The GC of the **Milky Way** is known to **nest a massive black hole** of mass of MBH ~ **4×10^6 M$_{\odot}$**

The RC is assumed to be expressed by a curve following the Newtonian potential of a point mass at the nucleus. and the rest of total mass is what is called dark matter---material that does not emit any light (a small fraction of it is ordinary matter that is too faint to be detected yet) but has a significant amount of gravitational influence. The total mass of the galaxy, M_g, including the extended dark halo, has been measured by analyzing the outermost RC and motions of satellite galaxies orbiting the galaxy, and the **mass up to ~100–200 kpc** has been estimated to be **3×10^{11} M$_{\odot}$.** ,

Mg = 0.3 Trillion Sun Masses

Where M$_{\odot}$ is the mass of Sun $M_{\odot} := 1.989 \cdot 10^{30} kg$ $M_g := 3 \cdot 10^{11} \cdot M_{\odot}$ $R_g := 8kpc$

Fit a Curve, (VFit), to the Milky Way Rotation Curve

$$V_{Fit} := ksmooth\left(r_g, V_{mwg}, 10\right)$$

Simple Model for Milky Way Galaxy that Approximates Galaxy Rotation Curves

Galactic Model: *Simple Model for Explaining Galaxy Rotation Curves*, A. Wojnar, Sporea

Model Parameters: M_0 the total galaxy mass, R_0 the observed scale length of the galaxy, r_c the core radius and fitting parameters b and β

Galactic Velocity Curve Fitting Model, vmw, with Five Fitting Parameters, Mg, R0, rc, b, and β

$M_{gas} := 10^{9.68} \cdot M_{\odot}$ $M_s := 10^{9.76} \cdot M_{\odot}$ $R_0 := 2.6kpc$ $r_c := 0.88kpc$ $b := 0.352$ $\beta := 1$

$M_g = 5.967 \times 10^{41} kg$ $M_{tot} := M_{gas} + M_s$ $X_M := \left(M_{gas} + M_s\right) \cdot M_g^{-1}$

$$v_{model}(r) := \sqrt{\frac{G \cdot M_{tot}}{r} \cdot \left(\sqrt{\frac{R_0}{r_c} \cdot \frac{r}{r + r_c}}\right)^{3\beta} \cdot \left[1 + b \cdot \left(1 + \frac{r}{R_0}\right)\right]}$$

$$v_{model}(20kpc) = 202.875 \cdot \frac{km}{s}$$

The Dark Halo Density profile:

DM Model: *Untied Rotation Curve of the Galaxy, Decomposition Bulge, Disk, Dark Halo*, Sofue

ρ_{hc} and R_h are constants giving the central mass density (ρ_{hc}) and scale radius of the halo, respectively

$\rho_{hc} := 0.03 \cdot M_{\odot} \cdot parsec^{-3}$ $\quad R_h := 5.5kpc$ $\quad light\_year := 0.000306kpc$

$\rho_{halo}(r) := \rho_{hc} \cdot \left[1 + \left(\frac{r}{R_h}\right)^2\right]^{-1}$ $\quad V_{inf} := \sqrt{4 \cdot \pi \cdot G \cdot \rho_{hc} \cdot R_h^{\,2}}$ $\quad V_{inf} = 221.258 \cdot \frac{km}{s}$

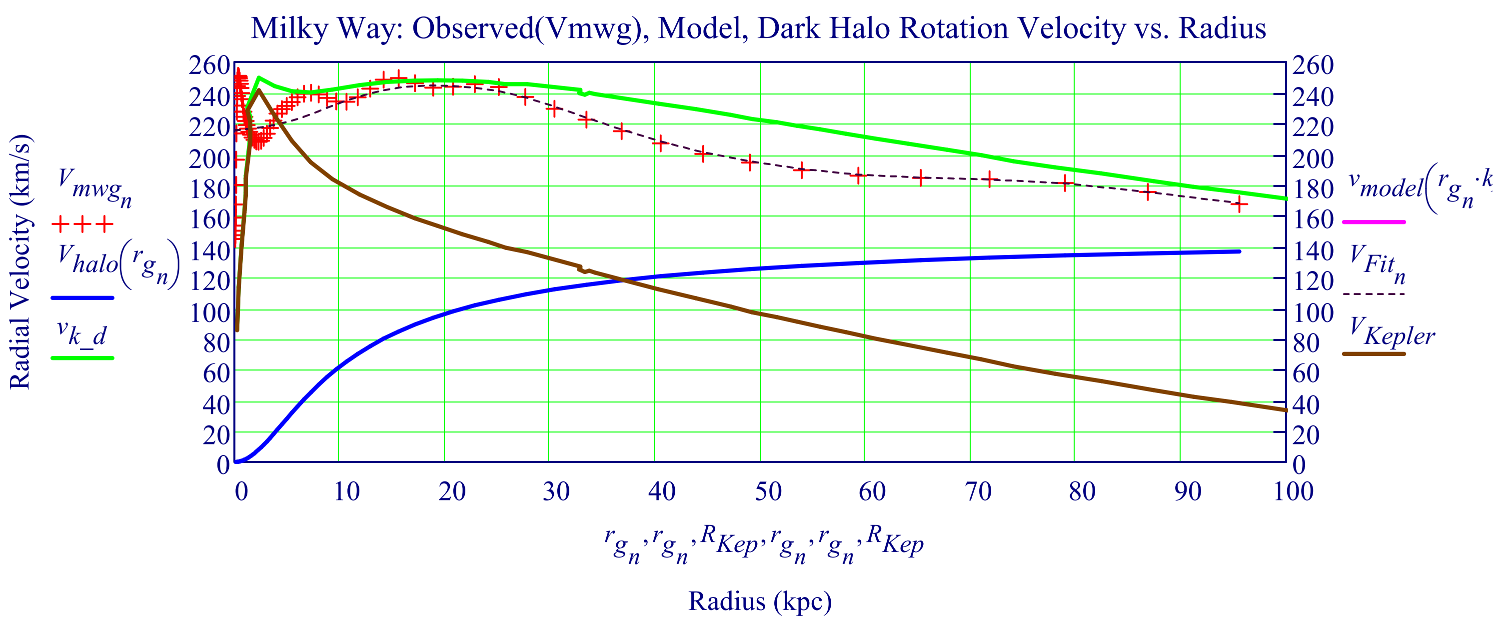
Estimate of Dark Halo - Isothermal Spherical Distribution

$V_{inf} := 150 \frac{km}{s}$ $\quad V_{halo}(r) := V_{inf} \cdot \left(1 - \frac{R_h}{r \cdot kpc} \cdot atan\left(\frac{r \cdot kpc}{R_h}\right)\right) \cdot \frac{1}{\frac{km}{s}}$

Sum of Keplerian and Dark Halo Distributions

$v_{k\_d} := V_{Kepler} + \overrightarrow{V_{halo}(R_{Kep})}$

Velocity Plots: Milky War Data (++), Vhalo of Dark Matter (Blue), vk_d Sum of Dark and Kepler, Galaxy Model (Purple), VFit Fit Curve to Data+ (Dashed Black), VKep Kepler Plot (Red)



Milky Way Galaxy Effects on Earth and Sun

The motion of the Sun (and the entire solar system) around Sagittarius A*—the supermassive black hole at the Milky Way's center—does have gravitational effects, though they're typically very small at our distance.

Orbital Motion of the Solar System

The Sun orbits Sagittarius A* at a distance of about 26,000 light-years (or ~8 kpc).
Orbital speed: ~220–230 km/s.
Orbital period: about 230 million years.

🌌 Local Gravitational Effects

☑ Tidal forces

The gravitational tidal forces from Sagittarius A* at this distance are tiny compared to local gravitational effects (like those from the Sun or nearby stars). They're not significant for: Planetary orbits

Solar system stability

☑ No measurable frame-dragging

The frame-dragging (Lense-Thirring) effect of Sagittarius A*'s spin is negligible at the Sun's location.

🌀 Overall Galactic Potential

While the black hole itself contributes to the galactic gravitational potential, it's only a small part of the total mass (~4 million solar masses out of ~100 billion solar masses for the galaxy).
Most of the solar system's orbital motion is due to the gravitational pull of the Milky Way's stars and dark matter halo, not the black hole alone.

🪐 Does it affect Earth or life?

No significant direct effects:
Tides, orbital perturbations, or frame-dragging from Sagittarius A* are negligible for the Earth.
The motion around the galaxy defines our cosmic neighborhood and velocities, but not daily or yearly physics on Earth.

✨ Summary

✔ The Sun's orbit around Sagittarius A* is real and well-documented, with spectacular implications for understanding galactic dynamics.
✔ Local gravitational effects on Earth (from Sagittarius A*) are incredibly small—no measurable impacts on the solar system's stability or everyday phenomena.

Gravitational Redshift / Blueshift (Cosmological Doppler Effect)

As the solar system orbits the galaxy's center:
Our motion relative to the CMB rest frame (~370 km/s) causes a dipole anisotropy in the CMB
— a measurable effect!
But locally (like within the solar system), the gravitational redshift due to Sagittarius A* is negligible because:

$M_{\odot} := 1.99 \cdot 10^{30} \cdot kg$ $M_{BH} := 200 \cdot 10^{6} \cdot M_{\odot}$

Negligible Effect

$r := 26000 \cdot LightYear$ $\frac{G \cdot M_{BH}}{c^2 \cdot r} = 0.012 \cdot 10^{-7}$

Angular Momentum $J := \frac{G \cdot \left(200 \cdot 10^{6} \cdot M_{\odot}\right)^2}{c}$

$J = 3.527 \times 10^{58} \frac{m^2 \cdot kg}{s}$

🌀 Frame-Dragging (Lense-Thirring Effect) Ω_{LT}

Sagittarius A* spins and drags spacetime around it (frame-dragging).

$\Omega_{LT} := \frac{2G \cdot J}{c^2 \cdot r^3}$

Effect on the Sun's orbit is: $M_{blackHole} := 200 \cdot 10^{6} \cdot M_{\odot}$

Negligible: $\Omega_{LT} = 3.519 \cdot \frac{rad}{s} \cdot 10^{-30}$