XXII. Λ-CDM Model Theory and Parameters

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See Section IIC: Table of The Hypothesized Thermal History of the Universe
See Section XXVI D: CMB Matter Power Spectrum
See Section XXXII: Some Key Problems of the ΛCDM Cosmology

Introduction

Planck temperature power spectrum multipole, ℓ

The discovery of the cosmic microwave background (CMB) by Penzias & Wilson (1965) established the modern paradigm of the hot ΛCDM cosmology. Almost immediately after this seminal discovery, searches began for anisotropies in the CMB – the primordial signatures of the fluctuations that grew to form the structure that we see today. This describes the cosmological parameter results from the Planck temperature power spectrum. This model is based upon a spatially-flat, expanding Universe whose dynamics are governed by General Relativity and whose constituents are dominated by cold dark matter (CDM) and a cosmological constant (Λ) at late times. The primordial seeds of structure formation are Gaussian-distributed adiabatic fluctuations with an almost scale-invariant spectrum. This model is described by only six key parameters. The focus is to investigate cosmological constraints from the temperature power spectrum measured by Planck. XXIII summarizes some important aspects of the **Planck temperature power spectrum**; we plot this as **$D \equiv \ell(\ell+1)C_L/2\pi$** (a notation we will use throughout this paper) versus multipole ℓ. The **temperature likelihood** used in this paper is a hybrid: over the multipole range $\ell = 2 - 49$, the likelihood is based on a component-separation algorithm applied to 91% of the sky. **See XXVI:** Calculation of CMB Multiple Moments Power Spectra, ℓ.

Λ-CDM Theoretical model

We shall treat anisotropies in the CMB as small fluctuations about a Friedmann-Robertson-Walker metric whose evolution is described by General Relativity. We parameterize the **mass fraction in helium by Y_P**. The process of standard big bang **nucleosynthesis** (BBN) can be accurately modeled, and gives a predicted relation between Y_P, the photon-baryon ratio, and the expansion rate (which depends on the number of relativistic degrees of freedom).

Ionization history - Optical Depth due to Reionization (Thomson Scattering), τ and Ionization Fraction, x_e

To make accurate predictions for the CMB power spectra, the background ionization history has to be calculated to high accuracy. Although the main processes that lead to recombination at **$z \approx 1090$** are well understood, cosmological parameters from Planck can be sensitive to sub-percent differences in the **ionization fraction x_e**. The process of recombination takes the Universe from a state of fully ionized hydrogen and helium in the early Universe, through to the completion of recombination with residual fraction $x_e \approx 10^{-4}$. Sensitivity of the CMB power spectrum to x_e enters through changes to the sound horizon at recombination, from changes in the timing of recombination, and to the detailed shape of the recombination transition, which affects the thickness of the last-scattering surface and hence the amount of small-scale diffusion (Silk) damping, polarization, and line-of-sight averaging of the perturbations. Cosmological parameters from Planck can be sensitive to sub-percent differences in the ionization fraction x_e.

The background recombination model should accurately capture the ionization history until the Universe is reionized at late times via ultra-violet photons from stars and or active galactic nuclei. We approximate reionization as being relatively sharp, with the **mid-point parameterized** by a **redshift z_{re}** (where $x_e = f/2$) the **Redshift of Half Reionization Width** parameter $z_{re} = 0.5$. Hydrogen reionization and the first reionization of helium are assumed to occur simultaneously, so that when reionization is complete $x_e = f = 1 + f_{He} \approx 1.08$, where **$f_{He}$ is the helium - to-hydrogen ratio by number**.

In this parameterization, the optical depth is almost independent of z_{re} and the only impact of the specific functional form on cosmological parameters comes from very small changes to the shape of the polarization power spectrum on large angular scales. The second reionization of helium (i.e., $He^+ \rightarrow He^{++}$) produces very small changes to the power spectra ($\Delta\tau \approx 0.001$, where τ is the optical depth to Thomson scattering) and does not need to be modeled in detail. We include the second reionization of helium at a fixed redshift of $z = 3.5$ (consistent with observations of Lyman- a forest lines in quasar spectra, e.g., Becker et al. 2011), which is sufficiently accurate for the parameter analyses described in this paper.

Initial conditions: Curvature Power Spectrum

In our baseline model we assume purely adiabatic scalar perturbations at very early times, with a (dimensionless) **Curvature Power Spectrum** parameterized by

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + (1/2)(dn_s/d\ln k)\ln(k/k_0)}$$

with **Scalar Spectrum Power-Law Index, n_s** and **Running of Spectral Index, $dn_s/d\ln k$** taken to be constant. For most of this paper we shall assume no "running", i.e., a power-law spectrum with $dn_s/d\ln k = 0$. The pivot scale, k_0, is chosen to be $\mathbf{k_0 = 0.05\ Mpc^{-1}}$, roughly in the middle of the logarithmic range of scales probed by Planck.

With this choice, n_s is not strongly degenerate with the **Amplitude Parameter A_s**.

The **Amplitude of the small-scale linear** CMB power spectrum is proportional to $e^{-2\tau} A_s$. Because Planck measures this amplitude very accurately there is a tight linear constraint between t and $\ln A_s$. For this reason we usually use $\ln A_s$ as a base parameter with a flat prior, which has a significantly more Gaussian posterior than A_s . A linear parameter re- definition then also allows the degeneracy between t and A_s to bewith n_s and $dn_s/d\ln k$ taken to be constant. For most of this paper we shall assume no "running", i.e., a power-law spectrum with $dn_s/d\ln k = 0$. The pivot scale, k_0 , is chosen to be $k_0 = 0.05\ \mathrm{Mpc}^{-1}$, roughly in the middle of the logarithmic range of scales probed by Planck. With this choice, n_s is not strongly degenerate with the amplitude parameter A_s to be explored efficiently. (The degeneracy between τ and A_s is broken by the relative amplitudes of large-scale temperature and polarization CMB anisotropies and by the non-linear effect of CMB lensing.) We shall also consider extended models with a significant amplitude of primordial gravitational waves (tensor modes). Throughout this paper, the (dimensionless) tensor mode spectrum is parameterized as a power-law with

$$\mathcal{P}_t(k) = A_t \left(\frac{k}{k_0}\right)^{n_t}$$

We define $r_{0.05} = A_t/A_s$, the primordial tensor-to-scalar ratio at $k = k_0$. Our constraints are only weakly sensitive to the tensor spectral index, n_t (which is assumed to be close to zero), and we adopt the theoretically motivated single-field indication consistency relation $n_t = -r_{0.05}/8$, rather than varying n_t independently. We put a flat prior on $r_{0.05}$, but also report the constraint at $k = 0.002\ \mathrm{Mpc}^{-1}$ (denoted $r_{0.002}$), which is closer to the scale at which there is some sensitivity to tensor modes in the large- angle temperature power spectrum. Most previous CMB experiments have reported constraints on Ratio of tensor primordial power to curvature power at $k_0 = 0.05\ \mathrm{Mpc}^{-1}$, $\mathbf{r_{0.05}}$

Power spectra

Over the last decades there has been significant progress in improving the accuracy, speed and generality of the numerical calculation of the CMB power spectra given an ionization history and set of cosmological parameters.

Base parameters

The first section of Table 1 lists our base parameters that have flat priors when they are varied, along with their default values in the baseline model. When parameters are varied, unless otherwise stated, prior ranges are chosen to be much larger than the posterior, and hence do not affect the results of parameter estimation.

Derived parameters: θ_{MC} Approximation to r_*/D_A (CosmoMC)

Matter-radiation equality z_{eq} is defined as the redshift at which $\rho_\gamma + \rho_\nu = \rho_c + \rho_b$ (where ρ_ν approximates massive neutrinos as massless). The redshift of last-scattering, z_*, is defined so that the optical depth to Thomson scattering from $z = 0$ (conformal time $\eta = \eta_0$) to $z = z_*$ is unity, (Redshift for which the optical depth equals unity) assuming no reionization. The optical depth is given by

$$\tau(\eta) \equiv \int_{\eta_0}^{\eta} \dot{\tau}\, d\eta'$$

where $\tau = -a\, n_e\, \sigma_T$ (and n_e is the density of free electrons and σ_T is the Thomson cross section). We define the **angular scale of the sound horizon at last-scattering**, $\theta_* = r_s(z_*)/D_A(z_*)$, where r_s is the sound horizon.

$$r_s(z) = \int_0^{\eta(z)} \frac{d\eta'}{\sqrt{3(1+R)}},$$

with $R \equiv 3\rho_b/(4\rho_\gamma)$.

Optical Depth of $r_* = 0.054$ means that about one CMB photon in 18 scatters from a free electron.

The parameter θ_{MC} (approximation to r*/D A (CosmoMC)) in Table 1 is an approximation to θ_* that is used in CosmoMC and is based on fitting formula given in Hu & Sugiyama (1996). Baryon velocities decouple from the photon dipole when Compton drag balances the gravitational force, which happens at $\tau_d \sim 1$, where

$$\tau_d(\eta) \equiv \int_{\eta_0}^{\eta} \dot{\tau}\, d\eta'/R.$$

Here, again, τ is from recombination only, without reionization contributions. We define a drag redshift z_{drag}, so that $\tau_d(\eta(z_{drag})) = 1$. The sound horizon at the drag epoch is an important scale that is often used in studies of baryon acoustic oscillations (BAO); we denote this as $r_{drag} = r_s(z_{drag})$.z_{drag} is the Redshift at which baryon-drag optical depth equals unity We compute z_{drag} and r_{drag} numerically from camb).

The characteristic wavenumber for damping, k_D, is given by

$$k_D^{-2}(\eta) = -\frac{1}{6}\int_0^{\eta} d\eta' \,\frac{1}{\dot{\tau}}\, \frac{R^2 + 16(1+R)/15}{(1+R)^2}$$

We define the angular damping scale, $\boldsymbol{\theta_D = p/(k_D D_A)}$*, where* D_A *is the comoving angular diameter distance to* z_*. *For our purposes, the normalization of the powe*r spectrum is most conveniently given by A_s. However, the alternative measure σ_8 is often used in the literature, particularly in studies of large-scale structure. By definition, σ_8 is the rms fluctuation in total matter (baryons + CDM + massive neutrinos) in 8 h^{-1} Mpc spheres at z = 0, computed in linear theory. It is related to the dimensionless matter power spectrum, $\mathcal{P}_m$, by

$$\sigma_R^2 = \int \frac{dk}{k}\, \mathcal{P}_m(k) \left[\frac{3 j_1(kR)}{kR}\right]^2$$

where R = 8 h^{-1} Mpc and j_1 is the **spherical Bessel function of order 1**. In addition, we compute $\Omega_m h^3$ (matter density $\Omega_m/\rho_{critical}$) a well-determined combination orthogonal to the acoustic scale degeneracy in flat models; see e.g., Percival et al. 2002 and Howlett et al. 2012), $10^9 A_s e^{-2t}$ (which determines the small-scale linear CMB anisotropy power), $r_{0.002}$ (the ratio of the tensor to primordial curvature power at k = 0.002 Mpc $^{-1}$), $\Omega_m h^2$ (the physical density in massive neutrinos), and the value of Y_P from the BBN consistency condition.

Acoustic scale

The characteristic angular size of the fluctuations in the CMB is called the acoustic scale. It is determined by the comoving size of the sound horizon at the time of last-scattering, $r_s(z_*)$, and the angular diameter distance at which we are observing the fluctuations, $D_A(z_*)$. With accurate measurement of seven acoustic peaks, Planck determines the observed angular size $\theta_* = r_s/D_A$ (CosmoMC) to better than 0.1% precision at 1σ :

$$\theta_* = (1.04148 \pm 0.00066) \times 10^{-2} = 0.596724^\circ \pm 0.00038^\circ$$

BAO surveys measure the distance ratio d_z

$$d_z = r_s\left(\frac{z_{drag}}{D_V(z)}\right) \qquad D_V(z) = \left[(1+z)^2 D_A^2(z)\frac{cz}{H(z)}\right]^{1/3}$$

The tight constraint on θ_* also implies tight constraints on some combinations of the cosmological parameters that determine D_A and r_s . The sound horizon r s depends on the physical matter density parameters, and D_A depends on the late-time evolution and geometry. Parameter combinations that fit the Planck data must be constrained to be close to a surface of constant θ_* .This surface depends on the model that is assumed. For the base Λ CDM model, the main parameter dependence is approximately described by a 0.3% constraint in the three-dimensional $\Omega_m - h - \Omega_b h^2$ subspace:

$$\Omega_m h^{3.2}(\Omega_b h^2)^{-0.54} = 0.695 \pm 0.002$$

Reducing further to a two-dimensional subspace gives a 0.6% constraint on the combination

$$\Omega_m h^3 = 0.0959 \pm 0.0006$$

Hubble parameter and dark energy density - Fixed Parameter: Matter Density Parameter, $\Omega_m h^3$

The Hubble constant, H_0 , and matter density parameter, Ω_m , are only tightly constrained in the Combination $\Omega_m h^3$ discussed above, but the extent of the degeneracy is limited by the effect of $\Omega_m h^2$ on the relative heights of the acoustic peaks. The projection of the constraint ellipse shown in onto the axes therefore yields useful marginalized constraints on H_0 and Ω_m (or equivalently Ω_Λ) separately. We find the 2% constraint on H_0 :

$H_0 = (67.4 \pm 1\,.\,4)$ km $^{s-1}$Mpc^{-1}. $\Omega_\Lambda = 0.686 \pm 0.020$ $\Omega_m h^2 = 0.1423 \pm 0.0029$

Optical depth - Reionication Optical Depth Parameter, τ

Small-scale fluctuations in the CMB are damped by Thomson scattering from free electrons produced at reionization. This scattering suppresses the amplitude of the acoustic peaks by $e^{-2\tau}$ on scales that correspond to perturbation modes with wavelength smaller than the Hubble radius at reionization. Planck measures the small-scale power spectrum with high precision, and hence accurately constrains the damped amplitude $e^{-2\tau A}$. With only unlensed temperature power spectrum data, there is a large degeneracy between t and A_s, which is weakly broken only by the power in large-scale modes that were still super-Hubble scale at reionization. However, lensing depends on the actual amplitude of the matter fluctuations along the line of sight. Planck accurately measures many acoustic peaks in the lensed tempera- ture power spectrum, where the amount of lensing smoothing depends on the fluctuation amplitude. Furthermore Planck's lensing potential reconstruction provides a more direct measurement of the amplitude, independently of the optical depth. The combination of the temperature data and Planck's lensing reconstruction can therefore determine the optical depth τ relatively well. The combination gives $\tau = 0.089 \pm 0.032$ (68%; Planck + lensing).
This provides marginal confirmation (just under 2 σ) that the total optical depth is significantly higher than would be obtained from sudden reionization at z ~ 6, and is consistent with the WMAP-9 constraint, $\tau = 0.089 \pm 0.014$, from large-scale polarization.

$$\tau_* = \int_{t_*}^{t_0} \Gamma(t)dt = c\sigma_e \int_{t_*}^{t_0} n_e(t)dt.$$

Each free electron has a cross-section $\boldsymbol{\sigma_e} = 6.65\ 10^{-29}$ m^2 for scattering with a photon given electron number density, $\boldsymbol{n_e}$, resulting in optical depth, $\boldsymbol{\tau_*}$.

Spectral index

The scalar spectral index (see below) is measured by Planck data alone to 1% accuracy: $n_s = 0.9616 \pm 0.0094$ (68%; Planck). Since the optical depth t affects the relative power between large scales (that are unaffected by scattering at reionization) and intermediate and small scales (that have their power suppressed by $e^{-2\tau}$), there is a partial degeneracy with n_s . Breaking the degeneracy between t and n_s using WMAP polarization leads to a small improvement in the constraint: $n_s = 0.9603 \pm 0.0073$. Comparing the two values of n_s, it is evident that the Planck temperature spectrum spans a wide enough range of multipoles to give a highly significant detection of a deviation of the scalar spectral index from exact scale invariance (at least in the base ΛCDM cosmology) independent of WMAP polarization information.

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0}\right)^{n_s - 1 + (1/2)(dn_s/d\ln k)\ln(k/k_0)}$$