II A. GLOSSARY: ΛCDM Model, Distance Ladder and Key Observational Probes

ΛCDM Model

H_0	Hubble constant today	km s^{-1} Mpc^{-1}
$H(z)$	Hubble parameter at reds z	km s^{-1} Mpc^{-1}
$a(t)$	Scale factor	dimensionless
z	Redshift	dimensionless
Ω_m	Total matter density fraction	dimensionless
Ω_b	Baryon density fraction	dimensionless
Ω_c	Cold dark matter density fraction	dimensionless
Ω_r	Radiation density fraction	dimensionless
Ω_Λ	Dark energy density fraction	dimensionless
Ω_k	Curvature density fraction	dimensionless
w	Dark-energy equation-of-state	dimensionless

Distances

Symbol	Meaning	Units
$D_C(z)$	Comoving distance	Mpc
$D_M(z)$	Transverse comoving distance	Mpc
$D_A(z)$	Angular diameter distance	Mpc
$D_L(z)$	Luminosity distance	Mpc
r_s	Sound horizon	Mpc
$D_V(z)$	Volume-averaged BAO distance	Mpc

Perturbations & Power Spectra

Symbol	Meaning	Units
k	Wavenumber	h Mpc^{-1}
$P(k)$	Matter power spectrum	(Mpc/h)^3
$T(k)$	Transfer function	dimensionless
n_s	Scalar spectral index	dimensionless
A_s	Primordial amplitude	dimensionless
σ_8	RMS fluctuations in 8 h^{-1} Mpc spheres	dimensionless

CMB Quantities

Symbol	Meaning	Units
ℓ	Multipole moment	dimensionless
C_ℓ	Angular power spectrum	μK^2
$D_\ell = \ell(\ell+1)C_\ell/2\pi$	Rescaled CMB spectrum	μK^2
τ	Optical depth to reionization	dimensionless
θ_s	Sound horizon angular scale	radians

Friedmann & Density Quantities

Symbol	Meaning	Units
ρ	Energy density	kg m^{-3}
ρ_c	Critical density	kg m^{-3}
$\rho_m, \rho_b, \rho_\Lambda$	Matter, baryon, dark-energy densities	kg

ΛCDM: The Standard Model of Cosmology

1. FLRW SPACETIME (Geometry)

Metric:

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

- $a(t)$: scale factor
- $k = -1, 0, +1$: curvature
- Spatial symmetry: homogeneous, isotropic

Vector Graphic Element:

A sphere grid expanding over time (3 concentric shells showing increased scale factor).

2. FRIEDMANN EQUATION (Dynamics)

$$H^2(a) = H_0^2[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda]$$

- Radiation dominates early universe
- Matter dominates intermediate
- Λ dominates today

Vector Graphic Element:

Stacked colored curves for each component vs. $a$or z.

3. ENERGY BUDGET OF THE UNIVERSE

Pie chart:

- **Dark Energy:** 68%
- **Dark Matter:** 27%
- **Baryons:** 5%
- **Photons + Neutrinos:** < 0.01%

(Values optional; use Planck 2018 best fit.)

Vector Graphic Element:

Clean pie graphic with 4 wedges.

4. EXPANSION HISTORY H(z)

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda}$$

Plot:

- x-axis: redshift z
- y-axis: $H(z)$line increasing toward early universe.

5. DISTANCES

Comoving Distance

$$D_C(z) = c\int_0^z \frac{dz'}{H(z')}$$

Angular Diameter Distance

$$D_A(z) = \frac{D_C(z)}{1+z}$$

Luminosity Distance

$$D_L(z) = (1+z)D_C(z)$$

Integrated Sachs–Wolfe Effect (ISW)

A contribution to the Cosmic Microwave Background (CMB) temperature anisotropy caused by time-varying gravitational potentials encountered by photons as they travel from the surface of last scattering to the observer. If gravitational potentials evolve—such as during radiation domination or late-time cosmic acceleration—photons experience a net energy shift, producing additional large-scale (low-ℓ) temperature anisotropy. The ISW effect provides an independent probe of dark energy through correlations between the CMB and large-scale structure.

$$\left(\frac{\Delta T}{T}\right)_{\rm ISW} = 2\int_{\eta_{\rm ls}}^{\eta_0} \dot{\Phi}(\eta)\, d\eta$$

The Sachs–Wolfe plateau corresponds to $10 \lesssim \ell \lesssim 30$.

where Φ is the Newtonian gravitational potential and η is conformal time.

Cosmological Significance:

The ISW effect enhances power at lowest multipoles of the CMB angular power spectrum and provides an independent observational probe of dark energy through correlations between CMB temperature maps and large-scale structure.

6. PRIMORDIAL POWER SPECTRUM

$$P(k) = A_s k^{n_s}$$

Nearly scale-invariant spectrum from inflation.

Vector Graphic Element:

Straight line with small tilt.

7. MATTER POWER SPECTRUM TODAY

$$P(k) = P_{\rm prim}(k)\, T^2(k)\, D^2(z)$$

- BAO Wiggles
- Turnover at equality

Vector Graphic Element:

P(k) curve with BAO oscillations.

8. CMB POWER SPECTRUM (TT)

Peaks correspond to:

- acoustic oscillations
- baryon loading
- photon diffusion
- matter–radiation ratio

Vector Graphic Element:

Classic TT multipole plot with labeled 1st, 2nd, 3rd peaks.

9. KEY OBSERVATIONAL INPUTS

Icons or mini-graphics for:

- **Type Ia SNe** — distance ladder
- **BAO** — standard ruler
- **CMB** — initial conditions
- **Weak Lensing** — structure growth
- **Galaxy Clustering** — P(k)

10. ΛCDM PARAMETER SET

The model is fully defined by **6 parameters**:

1. $\Omega_b h^2$
2. $\Omega_c h^2$
3. θ_s
4. A_s
5. n_s
6. τ

Cosmological Distances and the Distance Ladder

Cosmology uses both geometric distances defined within the FLRW model and an observational 'distance ladder' constructed from astrophysical standard candles and rulers. This section summarizes both perspectives.

1. Model-Based Cosmological Distances

- Comoving distance D_C(z): integral over H(z) that gives the line-of-sight comoving separation.

- Angular diameter distance D_A(z): relates physical size to observed angular size, important for BAO and CMB.

- Luminosity distance D_L(z): relates intrinsic luminosity to observed flux; key for supernova cosmology.

- Proper distance: scale-factor–dependent versions of the above, often used conceptually rather than directly observed.

2. Observational Distance Ladder

- Parallax: direct geometric distances to nearby stars ($\lesssim$ kpc).

- Cepheid variables: period–luminosity relation extends distances to nearby galaxies ($\lesssim$ tens of Mpc).

- Tip of the red giant branch (TRGB): standard-candle method complementary to Cepheids.

- Type Ia supernovae: standardized candles used out to $z \approx 1$–2 to measure D_L(z) and constrain cosmic acceleration.

- Surface brightness fluctuations, masers, and other secondary distance indicators refine local calibrations.

- BAO: a standard ruler (comoving sound horizon) imprinted in galaxy clustering, constraining D_A(z) and H(z).

Key Observational Probes

Cosmic Microwave Background (CMB): The CMB provides a snapshot of the Universe at recombination ($z \approx 1100$). Its temperature anisotropies encode information about the primordial fluctuation spectrum, the baryon and dark matter densities, the geometry, and the overall age and composition of the Universe. The acoustic peak pattern tightly constrains ΛCDM parameters.

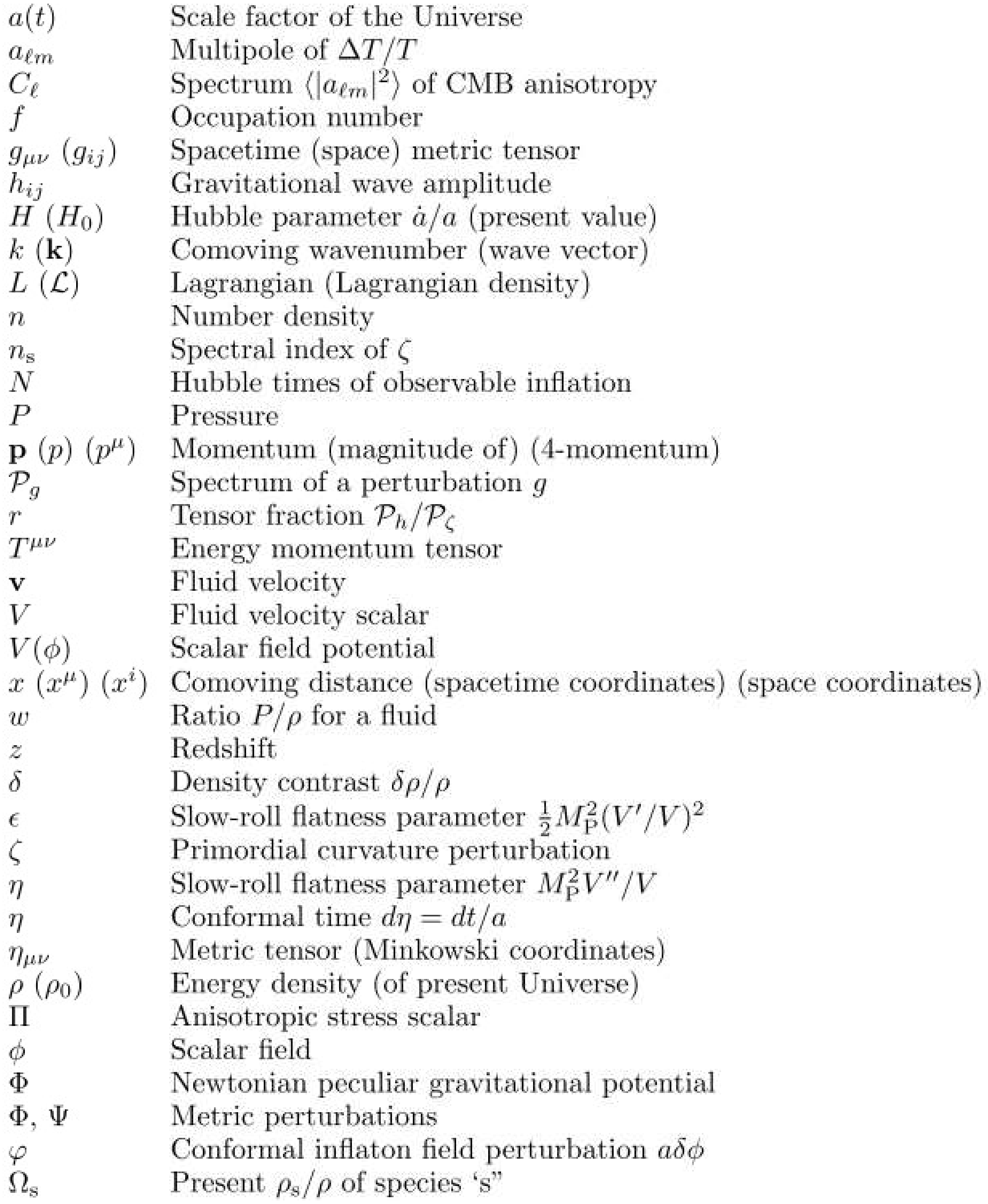
Baryon Acoustic Oscillations (BAO): BAO arise from sound waves in the early photon–baryon plasma, leaving a preferred comoving scale (the sound horizon) in the late-time distribution of galaxies. Measuring the BAO feature in galaxy clustering yields constraints on D_A(z) and H(z), providing a robust standard ruler.

Type Ia Supernovae (SNe Ia): Type Ia supernovae act as standardizable candles, allowing measurements of luminosity distance as a function of redshift. Their discovery in the late 1990s revealed that the cosmic expansion is accelerating, motivating dark energy/Λ.

Weak Gravitational Lensing: Weak lensing measures the coherent distortion of background galaxy shapes by intervening mass. It probes the projected matter distribution and the growth of structure, complementing geometry-based probes like BAO and SNe.

Galaxy Clustering and Redshift Surveys: The large-scale distribution of galaxies, measured by redshift surveys, constrains the matter power spectrum P(k), the bias between galaxies and dark matter, and the growth rate of structure. Redshift-space distortions further probe the velocity field and gravity on large scales.

More Detailed Cosmology Nonmenclature



$a(t)$	Scale factor of the Universe
$a_{\ell m}$	Multipole of $\Delta T/T$
C_ℓ	Spectrum $\langle \lvert a_{\ell m}\rvert^2\rangle$ of CMB anisotropy
f	Occupation number
$g_{\mu\nu}$ (g_{ij})	Spacetime (space) metric tensor
h_{ij}	Gravitational wave amplitude
H (H_0)	Hubble parameter $\dot{a}/a$ (present value)
k $(\mathbf{k})$	Comoving wavenumber (wave vector)
L $(\mathcal{L})$	Lagrangian (Lagrangian density)
n	Number density
$n_{\rm s}$	Spectral index of ζ
N	Hubble times of observable inflation
P	Pressure
$\mathbf{p}$ (p) (p^μ)	Momentum (magnitude of) (4-momentum)
$\mathcal{P}_g$	Spectrum of a perturbation g
r	Tensor fraction $\mathcal{P}_h/\mathcal{P}_\zeta$
$T^{\mu\nu}$	Energy momentum tensor
$\mathbf{v}$	Fluid velocity
V	Fluid velocity scalar
$V(\phi)$	Scalar field potential
x (x^μ) (x^i)	Comoving distance (spacetime coordinates) (space coordinates)
w	Ratio P/ρ for a fluid
z	Redshift
δ	Density contrast $\delta\rho/\rho$
ϵ	Slow-roll flatness parameter $\frac{1}{2}M_{\rm P}^2(V'/V)^2$
ζ	Primordial curvature perturbation
η	Slow-roll flatness parameter $M_{\rm P}^2V''/V$
η	Conformal time $d\eta = dt/a$
$\eta_{\mu\nu}$	Metric tensor (Minkowski coordinates)
ρ (ρ_0)	Energy density (of present Universe)
Π	Anisotropic stress scalar
ϕ	Scalar field
Φ	Newtonian peculiar gravitational potential
Φ, Ψ	Metric perturbations
φ	Conformal inflaton field perturbation $a\delta\phi$
$\Omega_{\rm s}$	Present $\rho_{\rm s}/\rho$ of species 's"

R_c	Radius Hubble Sphere (Region where galaxies recede subliminally)
$g_{\mu\nu}$	The Metric $g_{\mu\nu}$. A rank two symmetric tensor that encodes information about geometry.
$T_{\mu\nu}$	Einstein Stress-energy Tensor which describes matter and energy distributions.
$R^{\delta}{}_{\mu\nu}$	Riemann Tensor is a math construct used to characterize the curvature of space-time.
$R_{\mu\nu}$	The Ricci Tensor is a contraction of the Riemann Tensor.
R	The Ricci Scalar is a contraction of the Ricci Tensor.
$G_{\mu\nu}$	The Einstein tensor $G\mu\nu$ is defined in terms of the Ricci tensor and scalar.