XXIII B Python ΛCDM Six-Parameter Base Model - GCP

CAMB: Code for Anisotropies in the Microwave Background, **Spyder**: The Scientific Python Development Environment

Model Name: Flat ΛCDM (Six-parameter base model) Python 3.9, Spyder 5.1.5

This is the standard cosmological model used by Planck, WMAP, and CAMB if you set no special extensions. Specifically, this Python script computes a vanilla, spatially flat ΛCDM CAMB spectrum with no curvature, no evolving dark energy, and a single massive neutrino (0.06 eV).

Python Script Name: ***cmb_tt_planck_camb Dec 1 25.py***

Output from Python CMB Model: $\Lambda CDM := READPRN$("CMB_TT_model_scaled.csv")

Cosmological Parameters Used

Parameter	Value	Meaning
H_0	67.5 km/s/Mpc	Hubble constant today
Ω_b h²	0.022	Physical baryon density
Ω_c h²	0.122	Physical cold dark matter density
Ω_k	0	Curvature = 0 (flat universe)
m_v	0.06 eV	Single massive neutrino (minimal normal hierarchy)

- $h = H_0/100 = 0.675$
- $\Omega_b = \frac{\Omega_b h^2}{h^2} \approx 0.048$
- $\Omega_c \approx 0.268$
- Ω_ν (from 0.06 eV) ≈ 0.0014

→ Total matter: $\Omega_m \approx 0.317$

Reinization Parameter: 0.06

$$\Omega_\Lambda = 1 - \Omega_m - \Omega_r - \Omega_k \qquad \Omega_\Lambda \approx 1 - 0.317 - 0.0001 \approx 0.683$$

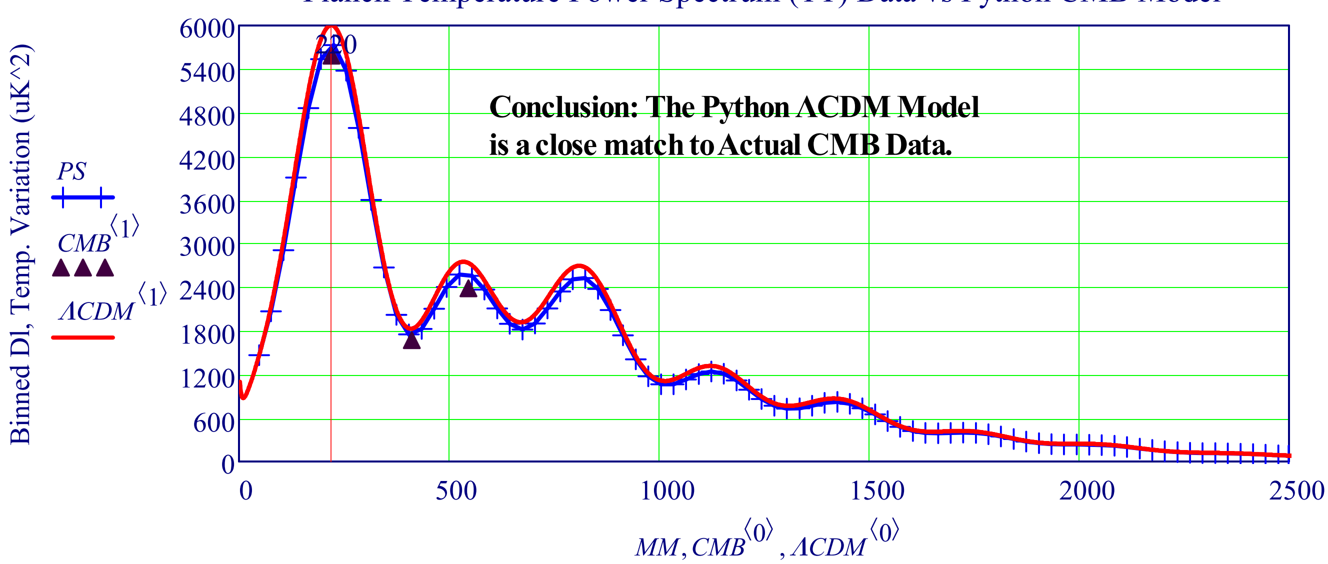
Parameter	Value	Meaning
A_s	2.1×10^{-9}	Primordial amplitude at pivot scale
n_s	0.965	Scalar spectral index (tilt)
Pivot scale (CAMB default)	k_pivot = 0.05 Mpc^{-1}	

CMB Calculations	
Setting	**Value**
lmax	2500
CMB_unit	'muK' (CAMB returns Dℓ in μK²)
lens_potential_accuracy	0 (lensing off for speed

Why These Parameters?

- close to Planck 2018 best-fit parameters,
- fast for CAMB to compute,
- stable and do not trigger unit or convergence errors,
- produce a first peak near ℓ ≈ 220.
- scale the model so that $D_{220} \approx 6000\ \mu K^2$,
- matching my desired normal

Planck Temperature Power Spectrum (TT) Data vs Python CMB Model



Multipole Moment, The Symbol for Multiple Moment is the letter "l"

Introduction for CAMB_CMB_TT and_Lensing

Python Program: CAMB_CMB_TT_and_Lensing_Example.py See: VXPhysics.com/Python

This program demonstrates how modern cosmology computes the cosmic microwave background (CMB) temperature anisotropy spectrum and **its modification by gravitational lensin**g using the Boltzmann code CAMB (Code for Anisotropies in the Microwave Background).

Script solves the coupled EinsteinBoltzmann equations for a flat ΛCDM cosmology to generate both unlensed and lensed CMB power spectra. Gravitational lensing by large-scale structure between the surface of last scattering and the observer redistributes CMB power, smoothing acoustic peaks & transferring power from small to slightly larger angular scales.

In temperature (TT) anisotropies, this lensing effect is subtle and **typically only a few percen**t, making it difficult to see on direct overlays. For this reason, the program also computes the CMB lensing potential power spectrum C_ℓ'3f^{φφ}, which directly encodes the projected matter distribution responsible for the deflections.

The outputs of this program illustrate three key ideas:

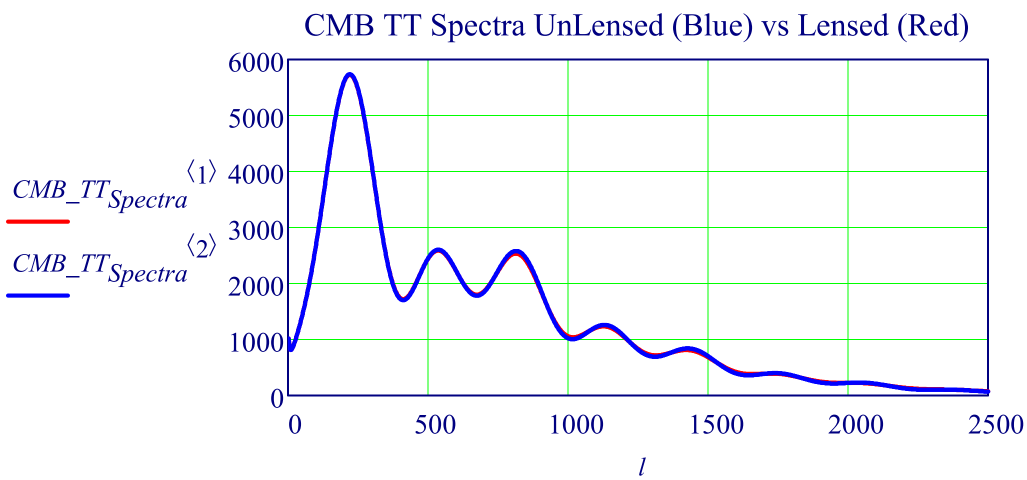
(1) the distinction between unlensed primordial anisotropies and lensed observed spectra,

(2) the physical role of gravitational lensing as a line-of-sight projection effect rather than a modification of early-universe physics, and

CAMB_CMB_TT_and_Lensing_Example.py

(3) the separation between directly observed quantities and model-dependent inferences in modern cosmology.

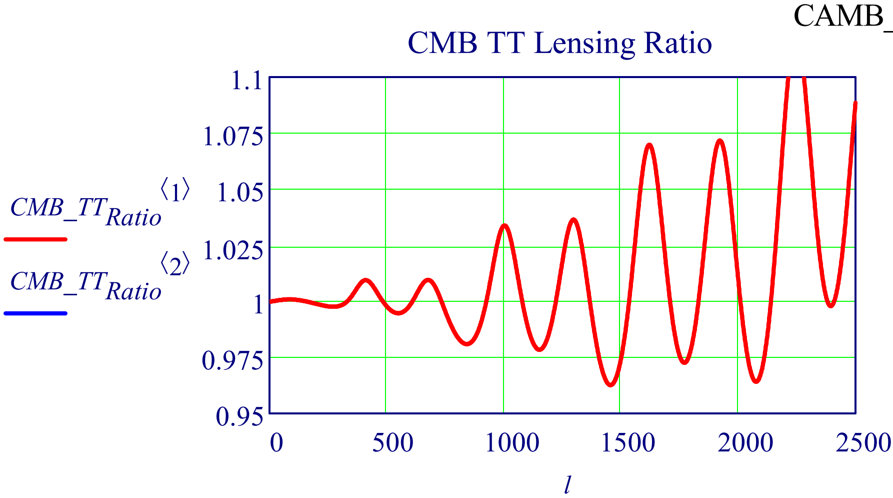
$CMB\_TT_{Spectra} := READPRN(\text{"cmb\_spectra\_TT\_EE\_Dl.csv"})$ $l := CMB\_TT_{Spectra}^{\langle 0 \rangle}$



The ratio of lensed to unlensed CMB power spectra isolates the effect of gravitational lensing, revealing percent-level smoothing in temperature anisotropies and a significantly stronger signature in polarization

$CMB\_TT_{Ratio} := READPRN(\text{"TT\_lensing\_ratio.csv"})$

CAMB_TT_EE_Ratio_to_CSV.py



Introduction for CAMB_EE_with_Planck_Overlay

This program compares theoretical predictions for the **CMB E-mode polarization (EE) power spectrum**, computed using CAMB, with observational data from the Planck 2018 mission on a single, self-contained vector-graphic page.

Unlike temperature anisotropies, **CMB polarization is more strongly affected by gravitational lensing.** Deflections caused by large-scale structure partially mix E- and B-modes, producing a clearer and more easily detectable lensing signature in the EE spectrum. As a result, EE polarization provides one of the cleanest observational confirmations of CMB lensing.

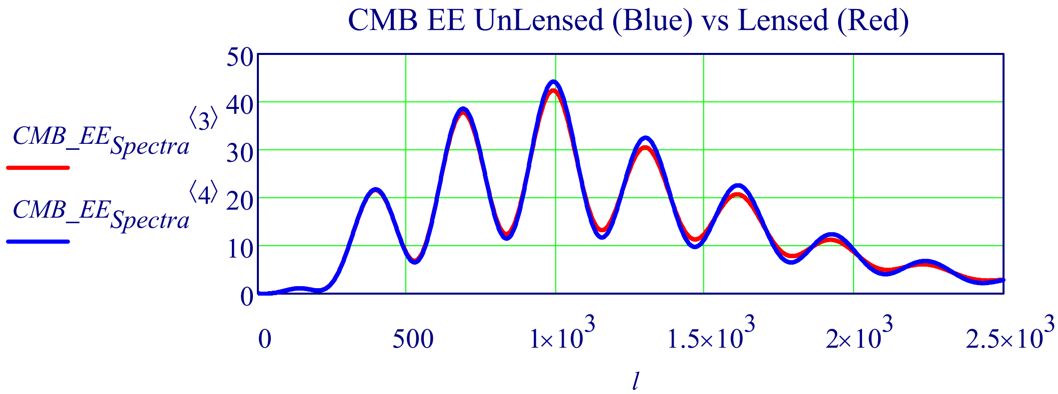
The script computes both unlensed and lensed EE spectra for a Planck-consistent ΛCDM cosmology and overlays these model predictions with binned Planck EE data. This directly illustrates how modern cosmological parameters are constrained by matching theoretical models to observed polarization power spectra.

The resulting single-page SVG figure emphasizes three fundamental points:
(1) gravitational lensing is required to match observed EE spectra,
(2) polarization data provide stronger evidence for lensing than temperature anisotropies alone, and
(3) cosmological conclusions arise from fitting physical models to data rather than from direct observation of underlying quantities such as dark matter or spacetime curvature.

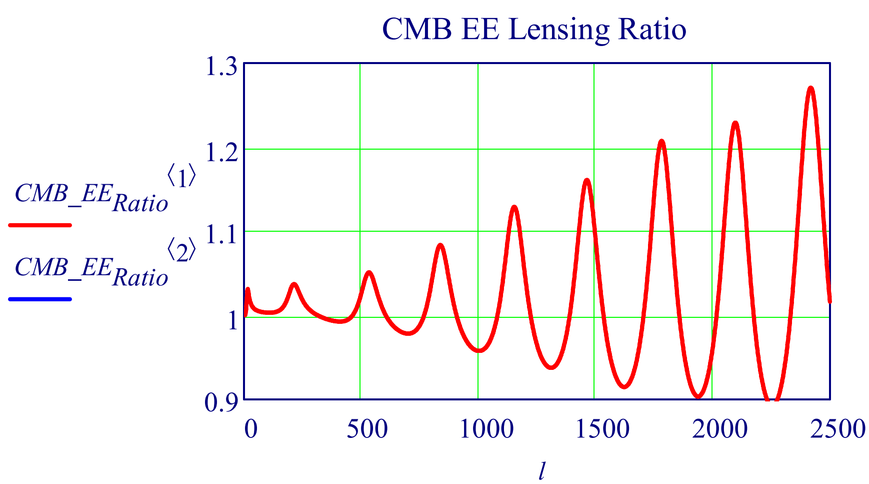
CAMB_CMB_TT_EE_and_Lensing_Example.py

$CMB\_EE_{Spectra} := READPRN(\text{"cmb\_spectra\_TT\_EE\_Dl.csv"})$ $l := CMB\_EE_{Spectra}^{\langle 0 \rangle}$

Dl_TT_lensed_muK2Dl_TT_unlensed_muK2Dl_EE_lensed_muK2Dl_EE_unlensed_muK2Dl_TE_lensed_muK2Dl_TE_unlensed_muK2



$CMB\_EE_{Ratio} := READPRN(\text{"EE\_lensing\_ratio.csv"})$ CAMB_TT_EE_Ratio_to_CSV.py



The CMB lensing potential power spectrum computed by CAMB

It is not a temperature or polarization spectrum;
it describes the projected gravitational potential that lenses the CMB.

What physical quantity is this?

Lensing potential $\phi(\hat{n})$

As CMB photons travel from the last-scattering surface to us, their paths are deflected by intervening large-scale structure. Those deflections are described by a scalar lensing potential ϕ

$$\vec{\alpha}(\hat{n}) = \nabla\phi(\hat{n})$$

- $\vec{\alpha}$ = deflection angle on the sky
- ϕ = line-of-sight integral of the gravitational potential

- Angular multipole ℓ
- Roughly corresponds to angular scale $\theta \sim \frac{180^\circ}{\ell}$

$C_\ell^{\phi\phi}$

- Power spectrum of the CMB lensing potential
- Dimensionless
- Encodes how much large-scale structure deflects CMB photons at each angular scale

$$D_\ell^{\phi\phi} = \frac{\ell(\ell+1)}{2\pi} C_\ell^{\phi\phi}$$

- A rescaled version used for plotting
- Makes power per logarithmic ℓ interval easier to see

How this differs from TT / EE / TE

Quantity	**What it describes**
TT	Temperature anisotropies at last scattering
EE	Polarization anisotropies
φφ	Gravitational lensing by intervening matt

TT and EE are affected by lensing
φφ is the lensing field itself

Why this spectrum is important

1. It directly traces matter

$C_\ell^{\phi\phi}$ **depends on:**

- Ωm
- growth of structure
- geometry of the Universe

Unlike TT:

- it is late-time
- it is sensitive to dark matter directly

2. It is what Planck actually reconstructs

Planck measures ϕ via:
4-point correlations of the CMB
then compares Cℓϕϕ to ΛCDM predictions

In harmonic space:

$$C_\ell^{\kappa\kappa} = \frac{\ell^2(\ell+1)^2}{4} C_\ell^{\phi\phi}$$

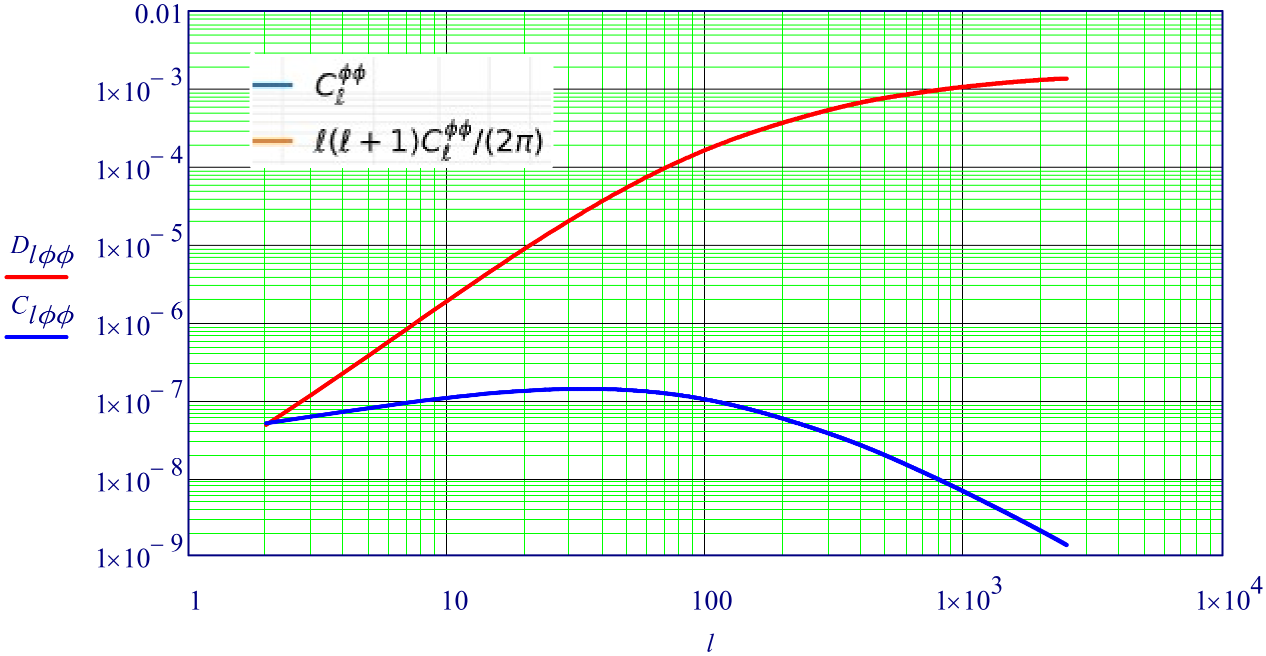
The CMB lensing potential power spectrum $C_\ell^{\phi\phi}$ quantifies the projected gravitational influence of large-scale structure on CMB photons and provides a direct probe of the matter distribution between recombination and the present epoch.

l, Cl_phi_phi, Dl_phi_phi

$\phi\phi := READPRN(\text{"lensing\_phi\_phi.csv"})$ $\quad DI := READPRN(\text{"cmb\_spectra\_Dl.txt"})$ $\quad l := \phi\phi^{\langle 0 \rangle}$

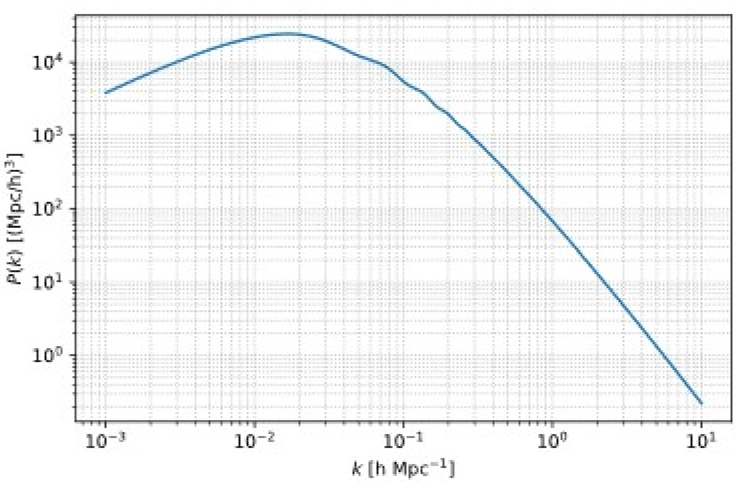
$$C_{l\phi\phi} := \phi\phi^{\langle 1 \rangle} \qquad D_{l\phi\phi} := \phi\phi^{\langle 2 \rangle}$$

CMB Lensing Potential, Cl phi phi, l(L+1)Clphi phi/2 pi

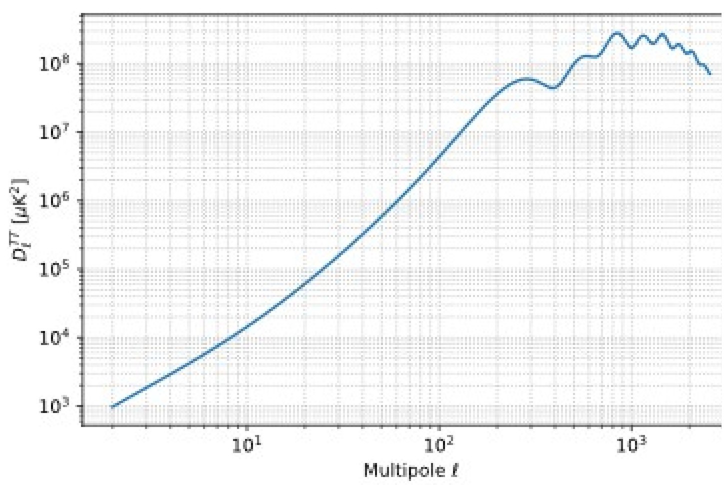


Cosmo LambdaCDB Model.py

lcdm_matter_Pk$_{camb}$



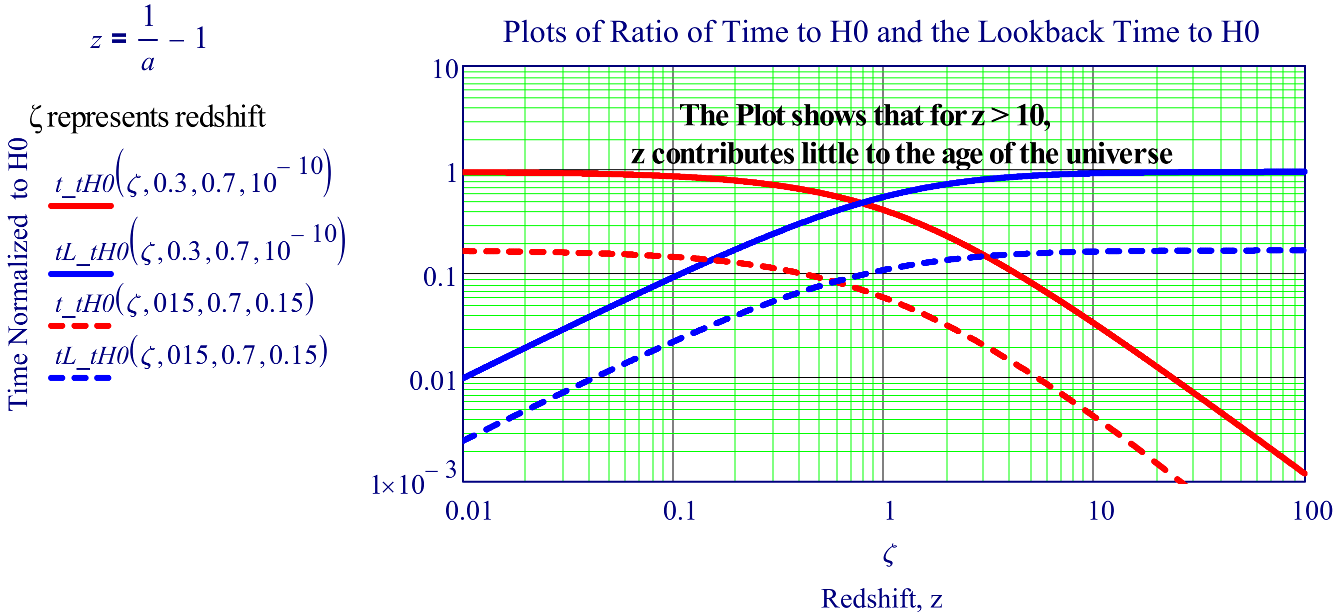
lcdm_CMB_TT_CAMB



Lookback Time versus Red Shift and Age of Universe (See Section VIII)

Plot from: VII. Equations and Values of Constants for Cosmological Parameters

$z = \frac{1}{a} - 1$



Evolution of the Hubble Factor:

Mass Conservation of non-relativistic matter implies $\rho_m \propto a^{-3} = (1+z)^3$.

In the ΛCDM model, dark energy is assumed to behave like a cosmological constant: $\rho_\Lambda \propto a^0 = (1+z)^0$.

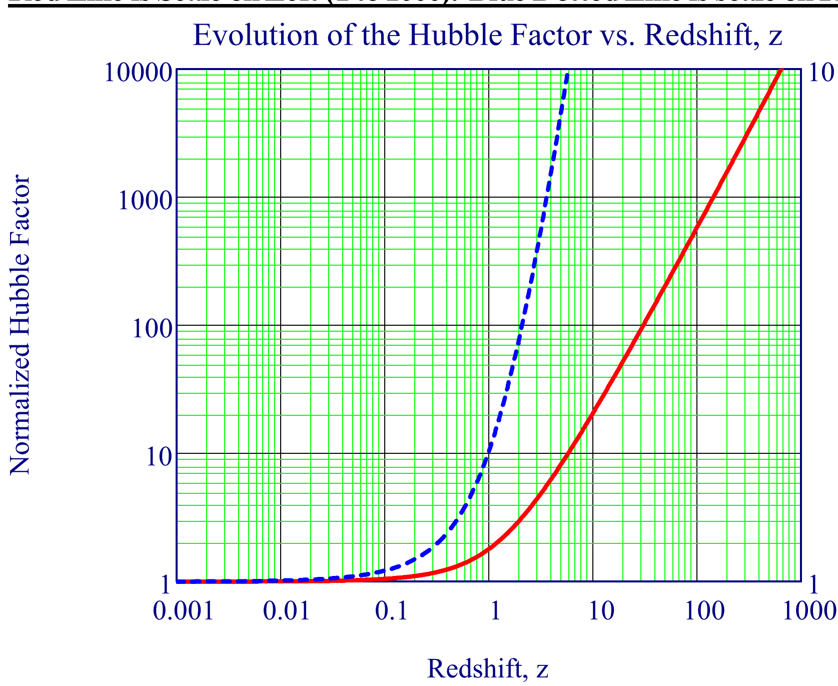
The density of radiation (and massless neutrinos) scales as $\rho_r \propto a^{-4} = (1+z)^4$ because the number density of photons is $\propto a^{-3} = (1+z)^3$ and the mass $E/c^2 = h\nu/c^2$ of each photon scales as $E \propto \lambda^{-1} \propto (1+z)^1 \propto a^{-1}$.

Dynamical Equation Specifying the Evolution of the Hubble Factor of Our Universe

$\Omega_{r0} := 8.7 \times 10^{-5}$

$$\frac{H}{H_0} = H\_H_0(z) := \sqrt{\Omega_{m0} \cdot (1+z)^3 + \Omega_{\Lambda 0} + \Omega_{r0} \cdot (1+z)^4}$$

Red Line is Scale on Left (1 to 1000). Blue Dotted Line is scale on Right (1 to 10)



Different z, Different Time, & Different Place

It must always be remembered that **different redshifts** correspond not only to **different times**, but also to **different places**. Thus, when we presume to connect observations of galaxies at different redshifts to derive an overall picture of cosmic evolution, we are implicitly assuming homogeneity; i.e. that "back-then, over there" is basically the same as "back-then, over here". For this to be true it is crucial that surveys for high-redshift galaxies contain sufficient cosmological volume to be "representative" of the Universe at the epoch in question. As we shall see, at $z > 5$ this remains a key challenge with current observational facilities.

Cosmology does not directly measure parameters

It measures patterns in light

from which physically meaningful parameter combinations are inferred.

CMB Lensing and Galaxy Weak Lensing: Complementary Probes of Structure

Gravitational lensing provides a unique window into the distribution of matter in the Universe by tracing the deflection of light by gravitational potentials along the line of sight. Two distinct but complementary realizations of this effect are widely used in modern cosmology: weak lensing of distant galaxies and lensing of the Cosmic Microwave Background (CMB). While both phenomena arise from the same underlying physics, they probe different epochs, scales, and sources of uncertainty.

Galaxy weak lensing measures the coherent distortion of background galaxy shapes caused by foreground large-scale structure. Because galaxies span a broad range of redshifts, galaxy lensing primarily probes the late-time growth of structure, making it highly sensitive to parameters such as Ωm, $\sigma 8$, and the growth factor D(z). In contrast, CMB lensing traces the cumulative deflection of photons originating from a single, well-defined source plane at redshift $z \simeq 1100$. As a result, CMB lensing integrates matter fluctuations over nearly the entire cosmic history, providing a clean and complementary probe of the matter distribution.

Although the observable quantities differ—shear correlations for galaxy lensing and reconstructed lensing potential for the CMB—the underlying theoretical description is closely related. In both cases, the lensing signal is governed by a line-of-sight projection of the matter power spectrum weighted by a lensing efficiency kernel. However, the redshift dependence of these kernels differs substantially, leading to different sensitivities to cosmological parameters and systematics.

The combination of galaxy and CMB lensing therefore offers a powerful internal consistency test of the ΛCDM model. Agreement between the two probes supports the standard picture of structure growth, while discrepancies may indicate observational systematics or new physics affecting the growth of perturbations.

What Cosmology Actually Measures

From Observables to Parameters: What Cosmology Really Measures

Modern cosmology is often presented as a discipline that “measures” fundamental parameters such as the matter density Ωm, the Hubble constant H0, or the amplitude of density fluctuations σ_8 . In practice, however, cosmological observations do not directly measure these quantities. Instead, t**hey measure statistical properties of observables**—angles, redshifts, brightness fluctuations, and correlations—which are then **mapped onto cosmological parameters through theoretical models.**

This indirect relationship between observation and inference is a defining feature of cosmology. Observables such as galaxy shapes, CMB temperature anisotropies, or galaxy clustering statistics depend on cosmological parameters in **coupled ways**, leading naturally to **parameter degeneracies**. As a result, experiments **often constrain** specific **combinations** of parameters rather than individual values. Well-known examples include the acoustic scale ***θs*** measured by the CMB and the parameter $S_8 = \sigma_8\sqrt{\Omega_m/0.3}$ured by weak lensing

Rather than being a limitation, these degeneracies encode valuable physical information. They reflect how geometry, expansion history, and structure growth interact in shaping the observable Universe. Understanding which combinations of parameters are constrained by a given observation clarifies both the strengths and the limitations of each cosmological probe.

This perspective highlights cosmology as an inverse problem: light propagates through spacetime, interacts with matter, and produces observable patterns on the sky. Cosmological models provide the forward map from parameters to observables, while data analysis seeks to invert this map to recover the underlying physical description