XXV. Mathematica CMBquick: Simulation of CMB Temperature Power Spectrum

WMAP Temperature Power Spectrum (TT) vs Multipole Moment Modeling

This Analysis is Based on Cyril Pitrou's Mathematica tools for creating CMB Spectra.

https://www2.iap.fr/users/pitrou/

"CMBquick is a package for Mathematica in which tools are provided to compute the spectrum and bispectrum of Cosmic Microwave Background (CMB)... CMBquick is a slow but precise and pedagogical, tool which can be used to explore and modify the physical content of the linear and non-linear dynamics. Second, its is a tool which can help developing templates for nonlinear computations, which could then be hard coded once their correctness is checked. The number of equations for non-linear dynamics is quite sizable and CMBquick makes it easy (but slow) to manipulate the non-linear equations, to solve them precisely, and to plot them."

Below are the results of CMBquick Simulation to find the Temp Power Spectrum for WMAP

Compare The Analysis Results Below to the Analysis from the Previous Section, XV

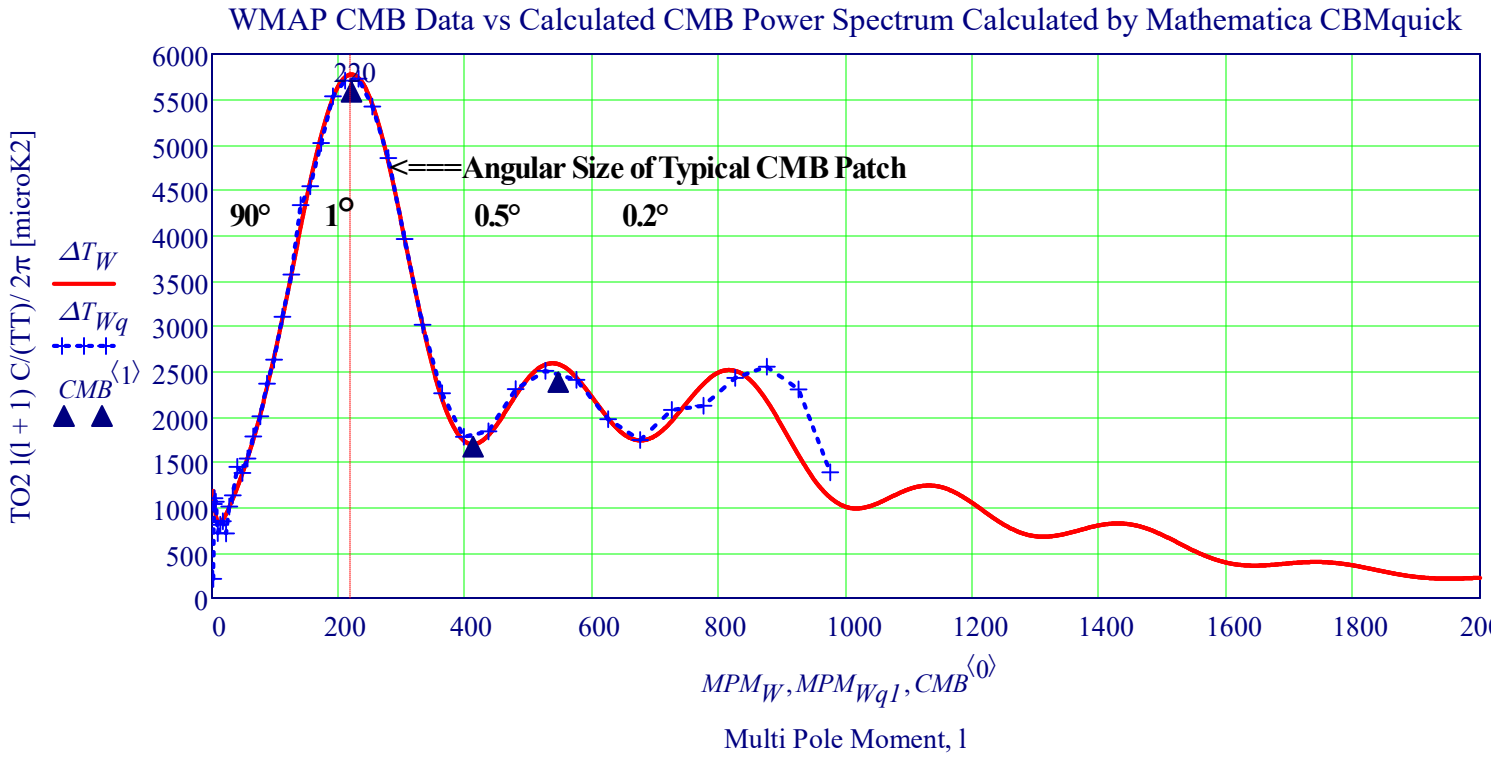
WMAP Temperature Power Anistrophies Calculated from Mathematica (CMB quick)

$WMAP_CMB := READPRN("CAMB_WMAP\text{-}CMBquick.dat")$

$WMAP_CMBq := READPRN("wmap_CMBq.dat")$

$\Delta T_W := WMAP_CMB^{\langle 1 \rangle}$ $\quad MPM_W := WMAP_CMB^{\langle 0 \rangle}$

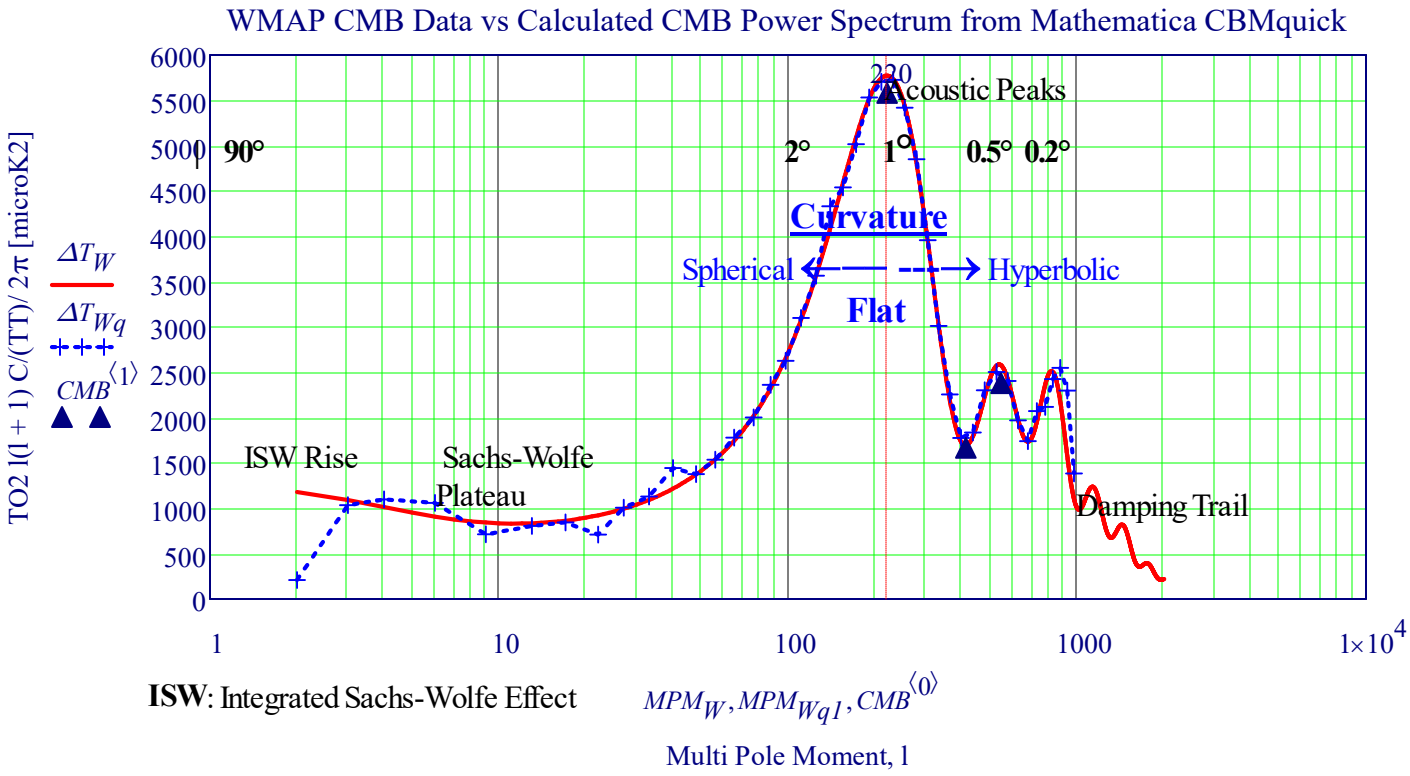
$\Delta T_{Wq} := WMAP_CMBq^{\langle 3 \rangle}$ $\quad MPM_{Wq1} := WMAP_CMBq^{\langle 0 \rangle}$



Angular Scale °, Curvature, and Projection Effects on CMB

The corresponding angle on the sky is approximately 100 / *l* degrees°

The Curvature of the Universe is Indicated by the Location of the First CMB Peak



ISW: Integrated Sachs-Wolfe Effect $MPM_W, MPM_{Wq1}, CMB^{\langle 0 \rangle}$

Projection Effects

See Pages 9, 10 and 11 for Definitions

$$\rho_0 = \frac{3 \cdot H_0^2}{8 \cdot \pi \cdot G} \qquad \rho_0 := 8.6443584621592 \cdot 10^{-27} \cdot \frac{kg}{m^3} \qquad \rho_{crit} = 1.879\ \ 10^{-29}\ h^2\ g\ cm^{-3}.$$

$$\Omega_M = \frac{8 \pi G \cdot \rho}{3 \cdot H_0^2} \qquad \Omega_\Lambda = \frac{\Lambda \cdot c^2}{3 \cdot H_0^2} \qquad \Omega_0 = 1 \text{ Radiation Transfer Function}$$

The mass is usually parameterized by Ω_0 which is the energy density in units of the critical density

$$\Omega_0 = \frac{\rho_T}{\rho_{crit}} \qquad \Omega_\Lambda = \frac{\rho_\nu}{\rho_{crit}} \qquad \Omega_0 + \Omega_\Lambda = 1$$

ρ_v is the vacuum contribution

$\Omega_0 > 0.1$ to 0.3 $\qquad \Omega_0 := 0.15$ $\qquad$ The Baryon Fraction is $\Omega_b h^2 = 0.01$ to 0.02

For the acoustic contributions, the k modes that reach extrema in their oscillation at last scattering form a harmonic series of peaks related to the sound horizon. This in turn is approximately

$$\frac{\eta_{star}}{\sqrt{1 + C \cdot (1 + R(n_{star}))}} \qquad R(n_{star}) = 30 \Omega_b \cdot h^2 \qquad C := \sqrt{3} - 1$$

Since $\Omega_b h^2$ must be low to satisfy nucleosynthesis constraints, the sound horizon will scale roughly as the particle horizon . The particle horizon at last scattering itself scales as

$$\eta_{star} = \left(\Omega_0 \cdot h^2\right)^{\frac{1}{2}} \cdot f_R \qquad f_R = \sqrt{\left[1 + \left(24\,\Omega_0 \cdot h^2\right)^{-1}\right]} - \sqrt{24\,\Omega_0 \cdot h^2}$$

CMBquick Cosmology CPLP Planck Perturbation Parameters

We compute the cosmology k dependent Boltzmann Hierarchy

Variable	Value	Units	
Ω_{b0}	0.049169		Abundance of baryons
Ω_{c0}	0.26474		Abundance of CDM
Ω_{r0}	0.000092414		Abundance of radiation (massless ? 's and photons)
$\Omega_{\Lambda 0}$	0.686		Abundance of Λ
Ω_K	0		Abundance of curvature
T_0	2.7255	K	Temperature of CMB
N_ν	3.045		Number of massless neutrinos
h	0.6727		Reduced H constant
τ_{rei}	0.079		Optical depth of reionization
n_s	0.9645		Scalar perturbations spectral index
k_{eq}	0.010362	Mpc^{-1}	k at equivalence time
z_{rei}	10.701		Redshift at reionization
z_{eq}	3395.7		Redshift at equivalence
z_{LSS}	1061.2		Redshift at t-trei = ln(2)
z_{dec}	1090.3		Redshift at max of visibility function
z_*	1091.2		Redshift at t-trei = 1
$d_A(z_*)$	13910.	Mpc	Angular distance at z *
$d_A(z_{eq})$	14078.	Mpc	Angular distance at equivalence
D_H	4456.56	Mpc	Hubble distance today
t_0	13.8308	Gyears	Age of the Universe
t_*	371 312.	years	Age of universe at z *
$r_{hor}(z_{dec})$	280.58	Mpc	Radius of horizon at z dec
η_0	14191.	Mpc	Conformal time today
A_s^2	2.4736×10^{-9}		Primordial scalar perturbations amp at k = 0.002 Mpc
n_S	0.9645		Scalar spectral index
r	0		Tensor to Scalar ratio at k = 0.002 Mpc
n_T	1		Tensor spectral index
σ_8	0.84516		Relies on extrapolation of the matter power spectrum