# XXVI A. Calculation of CMB Power Spectra from Model Parameters - CAMB Tool

**Code for Anisotropies in the Microwave Background [CAMB].**
**An Online CAMB Calculation Routine to calculate CMB_Model ΛCDM Model Parameters is available at:**
**https://lambda.gsfc.nasa.gov/toolbox/camb_online.html**

## Cosmological Model Parameters for Model Input

$\Omega$_b $h^2$ = 0.022600
$\Omega$_c $h^2$ = 0.112000
$\Omega$_ν $h^2$ = 0.000640
$\Omega$_Lambda = 0.724000
$\Omega$_K = 0.000000
$\Omega$_m (1-$\Omega$_K-$\Omega$_L) = 0.276000
100 θ (CosmoMC) = 1.039532
N_eff (total) = 3.046000
1 η, g= 1.0153 m_nu*$c^2$/k_B/T_nu0= 353.71 (m_nu= 0.060 eV)
Age of universe/GYr = 13.777
z* = 1088.75
r_s(z*)/Mpc = 146.38
100*θ = 1.039819
$z_{drag}$ = 1059.70
r_s($z_{drag}$)/Mpc = 149.01
k_D(z*) Mpc = 0.1393
100*θ_D = 0.160248
z_EQ (if v_nu=1) = 3216.47
100*θ_EQ = 0.847737
τ_recomb/Mpc = 284.72 τ_now/Mpc = 14362.3

## Fake Model Params for Comparison

**$\Omega$_b $h^2$ = 0.05**
$\Omega$_c $h^2$ = 0.112000
$\Omega$_ν $h^2$ = 0.000640
$\Omega$_Lambda = 0.724000
$\Omega$_K = 0.000000
$\Omega$_m (1-$\Omega$_K-$\Omega$_L) = 0.276000
100 θ (CosmoMC) = 1.039532
N_eff (total) = 3.046000

### Fake Model CMB Curve

$CMB\_Model_{Fake} := READPRN$("Lensedcls-CMB Spectrum Om_b h2 050.txt")

$\Delta T2K_{Fake} := CMB\_Model_{Fake}^{\langle 1 \rangle}$

$MPM_{Fake} := CMB\_Model_{Fake}^{\langle 0 \rangle}$

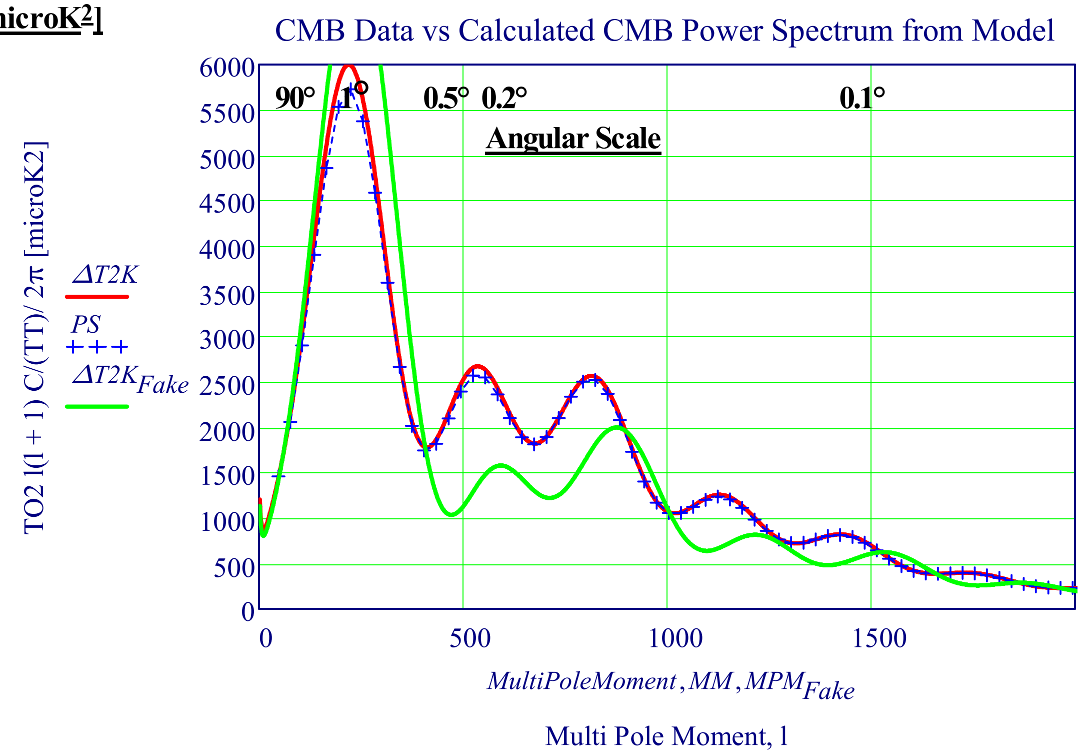
## Use the Online CAMB Calculation Routine with Above CMB Parameters ==> ΛCDM Model

$CMB\_Model := READPRN$("Lensedcls-CMB Spectrum.txt") $rows(CMB\_Model) = 2099$

$\Delta T2K := CMB\_Model^{\langle 1 \rangle}$ $MultiPoleMoment := CMB\_Model^{\langle 0 \rangle}$

**Note: The Excellent Match Between Data and the Model**

**TO² $l(l+1)$ C/(TT)/ 2π [microK²]**



# XXVI B. CMB Spherical Bessel Projection Approximation

Mathcad "Toy Model" To Investigate Use of Spherical Bessel Functions to Project 3D to 2D Sky

$$j_\ell(k(\eta_0 - \eta))$$

**The spherical Bessel functions project these 3D oscillations onto the 2D sky.**

These Bessel functions, ***$js_l$*** naturally have oscillatory behavior.

$$C_\ell^{TT} \approx \int dk\, k^2\, S_k^2\, j_\ell^2(kr)$$

So, in our **Approximate Delta-Function Model:**

The oscillations ***in $C_\ell^{TT}$*** directly come from the **Bessel functions**, ***$j_\ell$*** and the source function's own oscillations.

In this approximation the source function is static in time—it's treated as if the entire contribution to the CMB anisotropy is **from a single conformal time (last scattering)**. This is known as the "Delta-Function Approximation": the time integral in the projection of **$\Theta_l(k)$** is replaced by a single time (like a delta function at $\eta_*$).

A Sachs_Wolfe term was added given by the SW_plateau correction for Multipole Moments < 135.

Artifacts are a Possibility in Our Approximation. In real physics, the source function is not a delta function—it's spread over time, which smooths out some oscillatory features at high ℓ

It arises from **acoustic oscillations of the photon-baryon plasma** before recombination.

**Spherical Bessel Function, js(m,z). This is a 1200 x 500 Multipole by k Table of Bessel Function Values**

Factor to Normalize TT Spectrum ~ Unity

$\eta_0 := 1.4\cdot 10^4$ $\quad r := \eta_0$ $\quad x := 0..1199$ $\quad \ell_x := x$ $\quad Normalization := 2.2\cdot 10^{-11}$

$n := 0..499$ $\quad ell\_values := \ell$ $\quad k\_values_n := 0.0001 + 0.0006\cdot n$ $\quad max(k\_values) = 0.3$

$n\_ell := 1200$ $\quad n\_k := 500$ $\quad r := 14000$ $\quad$ Sachs-Wolfe plateau amplitude: $SW\_plateau := 1.9\cdot 10^{-13}$

**Generate Spherical Bessel Functions j-ell(k * r)**

```
SBF := | for ℓ ∈ 0..1199
       |   for j ∈ 0..499
       |     BM[ℓ,j] ← js(ℓ, k_values[j]·14000)
       | BM
```

**Source function for all k - Delta-Function Approximation**

$$S\_k(k) := k^{0.2}\, exp\left(\frac{-k}{0.1}\right)\cdot \left|sin(0.02\cdot k\cdot r)\right|$$

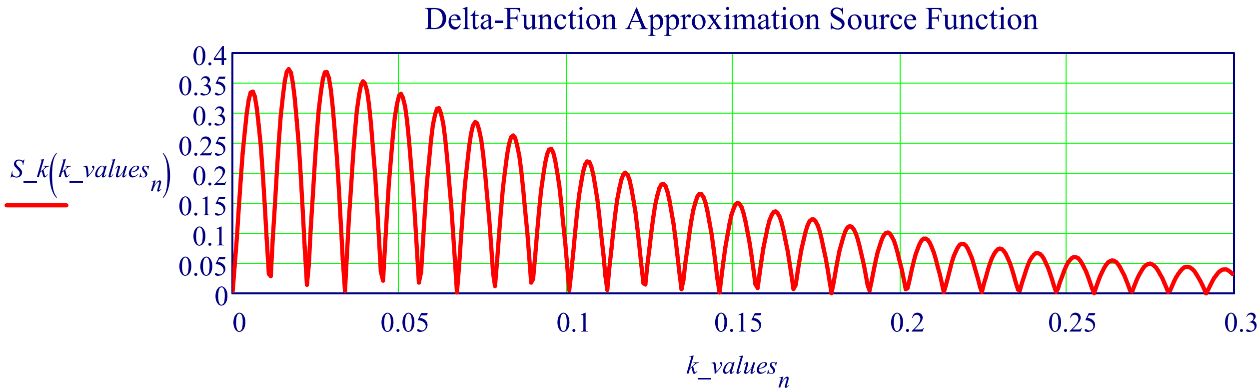
$n\_k := length(k\_values)$

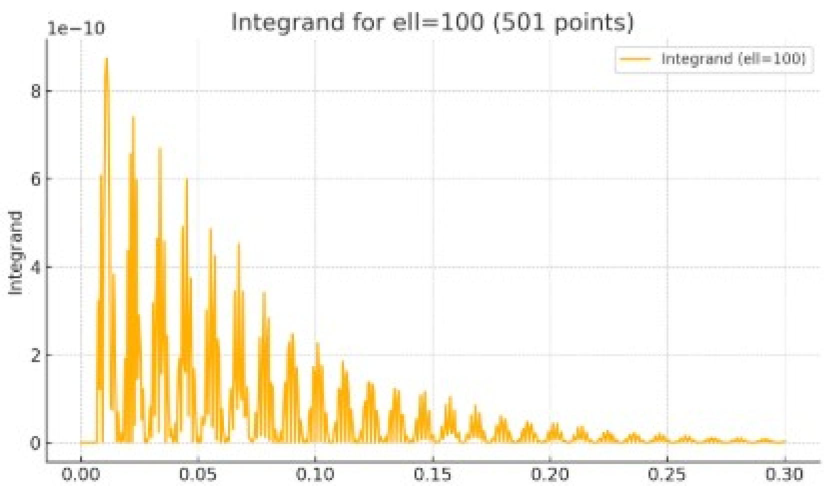
$S\_vector_n := S\_k(k\_values_n)$

**Compute approximate TT spectrum for each Multipole ℓ and Integrate**

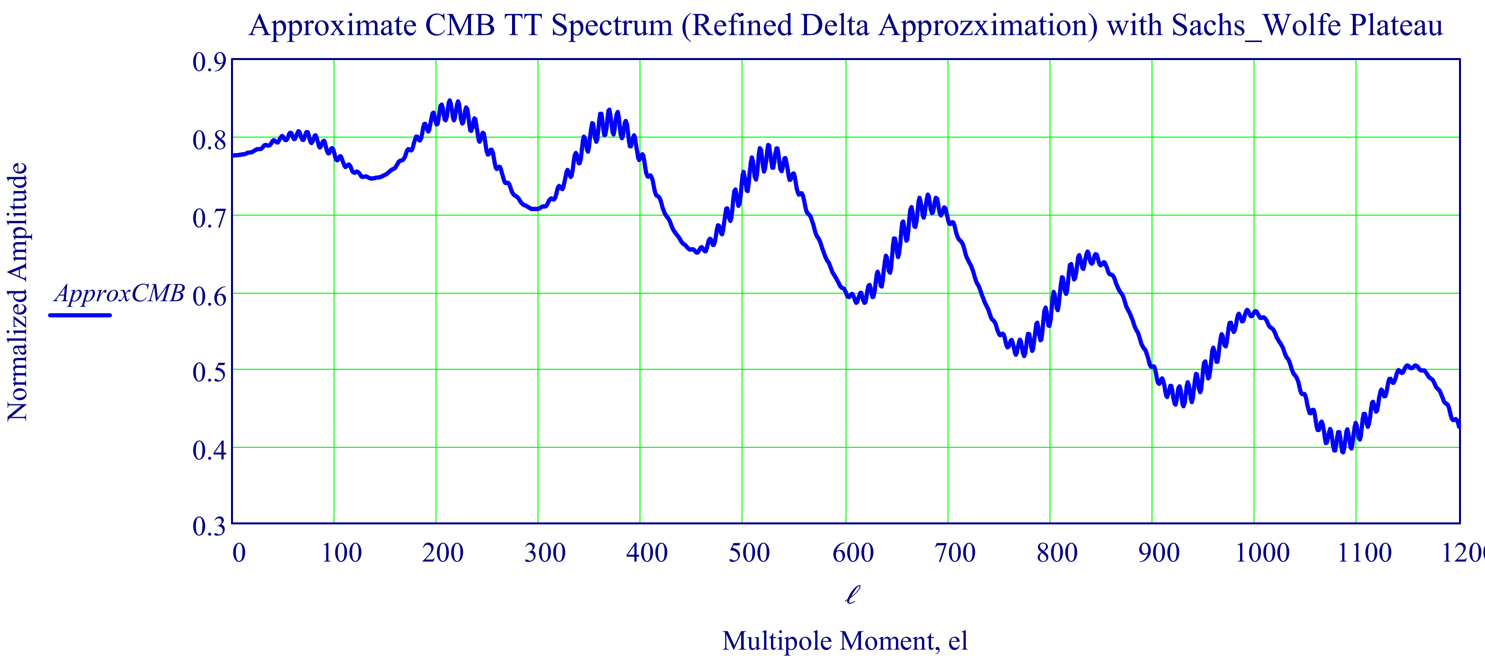
```
TT_Spectrum := | for ℓ ∈ 0..1199
               |   | for j ∈ 0..499
               |   |   integrand[ℓ,j] ← (k_values[j])^2·(S_vector[j])^2·(SBF[ℓ,j])^2
               |   | h ← k_values[1] − k_values[0]
               |   | sum ← 0
               |   | for j ∈ 0..499
               |   |   sum ← sum + integrand[ℓ,j]
               |   | TTS[ℓ] ← (h/2)·(integrand[ℓ,0] + 2·sum + integrand[ℓ,499])
               |   | ℓ ← TTS[ℓ] + SW_plateau  if ℓ < 135
               | TTS / Normalization
```

$ApproxCMB := ksmooth(\ell, TT\_Spectrum, 6)$




**Spherical Bessel Function Projection Approximation to CMB TT Amplitude Multipole Distribution**




Reference: *SMALL SCALE COSMOLOGICAL PERTURBATIONS: AN ANALYTIC APPROACH,* Wayne Hu, arXiv:astro-ph/9510117v2

# XXVI C. CMB TWK Empirical Gaussian TT Power Spectra Model

**Features of This Model**

The primordial seeds of structure formation are Gaussian-distributed adiabatic fluctuations with an almost scale-invariant spectrum. This model uses Gaussian Waveforms for each acoustic peak (localized power) and has an Excellent Match to Planck TT spectrum shape up to ℓ ~ 2400,

**TT Power Spectrum Plot Data Sources**

Planck 2018 TT temperature power spectrum

Planck 2018 results. https://lambda.gsfc.nasa.gov/education/lambda_graphics/more/tt_spec

$C := READPRN(\text{"Planck CMB TT Spectra3.txt"})$ $\quad \ell := C^{\langle 0 \rangle}$ $\quad D_{data} := C^{\langle 1 \rangle}$ $\quad rows(\ell) = 111$

**First Peak:** $max(D_{data}) = 5793.4395$ $\quad match(max(D_{data}), D_{data}) = (34)$ $\quad \ell_{34} = 225.2$

**First Peak:** Corresponds to the largest acoustic oscillation in the photon-baryon plasma before recombination. The sound horizon—the maximum distance a sound wave (photon-baryon density wave) could travel in the early universe before photons decoupled from baryons (recombination). Sound Horizon at recombination $r_s$ ~ 144Mpc

**Second Peak:** Second harmonic of acoustic oscillations of in the photon-baryon plasma. Maximum rarefraction.

Ratio of Peak Locations: $\frac{225}{538} = 0.418$ gives $\Omega_b h^2 \sim 0.022$

**TWK Cosmological parameters to Match the Planck 2018 TT best-fit approximations**

Multipole Amplitude Dispersion, Multipole Amplitude Dispersion, Multipole Amplitude Dispersion Multipole Amplitude Disp

$$P12 := \left(225.2\ \ 5.793\times10^3\ \ 95\ \ 538\ \ 2.5\times10^3\ \ 90\ \ 810\ \ 2.5\times10^3\ \ 100\ \ 1.1\times10^3\ \ 1.2\times10^3\ \ 100\right)^T$$

$$P9 := \left(1.4\times10^3\ \ 850\ \ 120\ \ 1.7\times10^3\ \ 300\ \ 130\ \ 2\times10^3\ \ 200\ \ 200\right)^T \qquad P := stack(P12, P9) \qquad rows(P) = 21$$

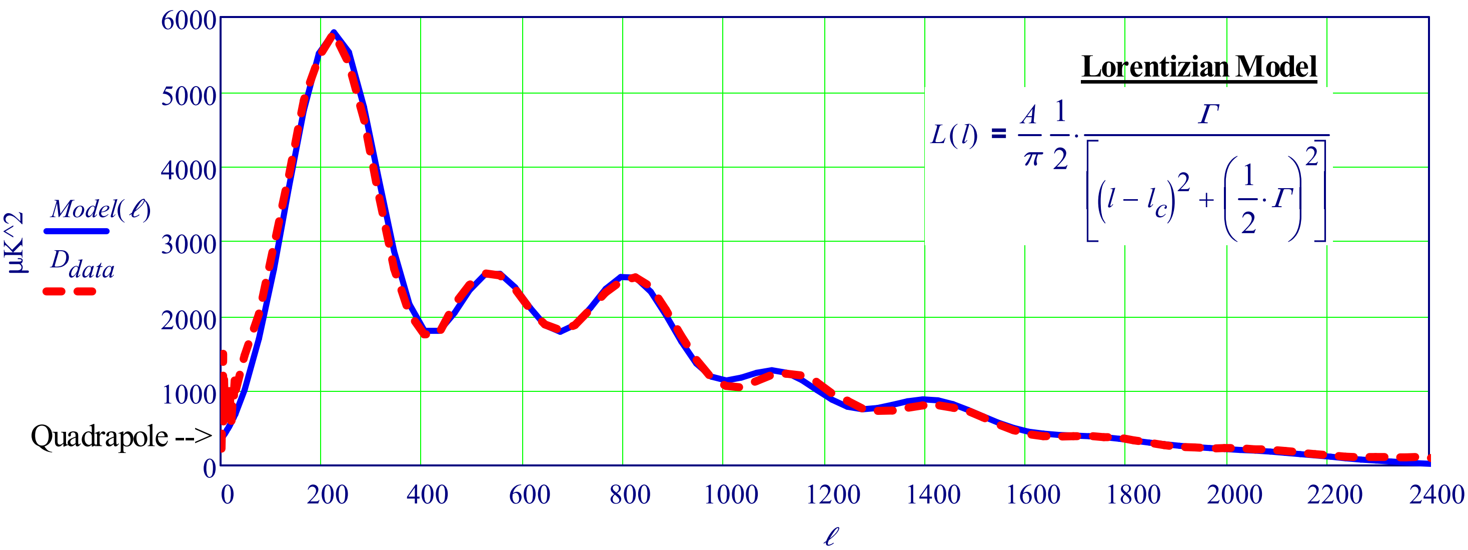
**Gaussian TT Mathcad Model Function for Single Peak**

$$Gauss(\ell, i) := P_{1+3\cdot i}\cdot exp\left[-\left[\frac{\left(\ell - P_{0+3i}\right)^2}{2\cdot\left(P_{2+3\cdot i}\right)^2}\right]\right]$$

**Seven Peak Gaussian Composite Model**

$$Model(\ell) := \sum_{i=0}^{6} Gauss(\ell, i)$$

Excellent CMBR TWK Gaussian Empirical TT Power Spectra Match to Planck Data (Red).




Multipole Moment, ℓ