XXVI D. CMB Matter Power Spectrum

See XXII. Λ-CDM Model Theory and Parameters

Current Universe Matter Power Spectrum: Amplitude vs Scale

Linear Matter Power Spectrum at z = 0: P(k) vs. k in Log-Log Space

Key points about this plot

- The horizontal axis is wavenumber k(units typically $h\,\mathrm{Mpc}^{-1}$).
- The vertical axis is the power spectrum $P(k)$(units typically $(h^{-1}\mathrm{Mpc})^3$).
- On large scales (small k), $P(k)$rises roughly like a power law; on smaller scales (large k) it drops more steeply. For example, the asymptotic behaviours are approximately $P(k) \propto k^{n_s}$at very small $k$and $P(k) \propto k^{n_s-4}$at large $k$for a matter-dominated epoch (see e.g., the "linear matter power spectrum" discussion). NASA/IPAC Extragalactic Database+2Wikipedia+2
- A characteristic "turn-over" occurs at the scale corresponding to matter–radiation equality (typically $k_{\mathrm{eq}} \sim 2 \times 10^{-2}\ h\,\mathrm{Mpc}^{-1}$). Wikipedia+1

The Matter Power Spectrum

$\boldsymbol{P(k)}$**vs.** $\boldsymbol{k}$

- Horizontal axis: **comoving wavenumber** $k[h/\mathrm{Mpc}]$
- Vertical axis: **matter density fluctuation amplitude** $P(k)$
- Epoch: **today** (redshift $z = 0$)
- Physics it probes:
  - growth of **large-scale structure**
  - horizon scale at matter–radiation equality
  - dark matter clustering
  - small-scale suppression from transfer functions
- Typical shape:
  - **Power-law rise** at large scales (small k)
  - **Turnover peak** near $k \sim 0.02\,h/\mathrm{Mpc}$
  - **Decay** at small scales (large k)

This tells us how **matter is distributed today**.

How the Two Spectra Are Related

Both measure the same initial primordial power spectrum

$$P_{\mathrm{prim}}(k) \propto k^{n_s}$$

but evolve it differently:

Quantity	Domain	Epoch	Sensitive to
CMB C_ℓ	angular multipoles ℓ	$z = 1100$	photon–baryon sound waves
Matter $P(k)$	spatial wavenumber k	$z = 0$	growth of structure under gravity

They are related by the **radiation transfer function** and **projection from 3D→2D sky**:

$$C_\ell = 4\pi \int \frac{dk}{k} P(k)\, |\Delta_\ell(k)|^2$$

This means:

- The **CMB is a 2D angular projection** of primordial 3D fluctuations.
- The **matter spectrum is the 3D distribution** after ~13.8 Gyr of growth.

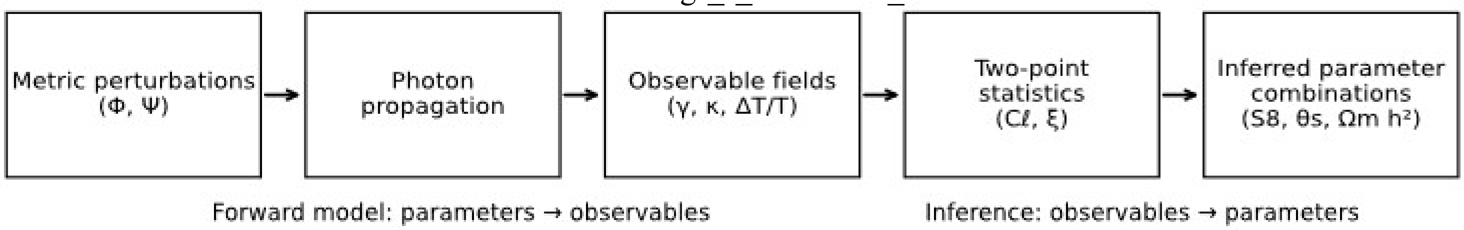
CMB Angular Power Spectrum
Multiple peaks
Each peak encodes harmonic sound modes in the early universe
High-ℓ damping tail
Low-ℓ plateau

What is Labeled:

- Physics & geometry → photon propagation
- Observables: $\gamma, \kappa, \Delta T/T$
- Inferred parameters: S_8, Ω_m, θ_s

Information Flow Diagram

fig7_1_information_flow



Kernels vs Transfer Functions (CAMB/Class)

In modern cosmology, predicted observables are constructed in two distinct steps: evolution and projection. These steps are encoded mathematically by transfer functions and kernels, respectively.

Transfer functions describe how primordial perturbations evolve with time and scale:
d(k, z) = T(k, z) d_prim(k)

Boltzmann codes such as CAMB and CLASS compute these transfer functions by solving the linearized Einstein–Boltzmann equations for matter, radiation, and metric perturbations. The resulting matter power spectrum is
$P(k, z) = P\_prim(k)\, T^2(k, z)$.

Kernels project evolved perturbations into observables by weighting contributions along the line of sight. For weak gravitational lensing, the convergence field is given by $\kappa(\hat{n}) = \int W(\chi)/\chi^2\ \delta(\chi(\hat{n}, z))\, d\chi$.

In harmonic space, kernels appear explicitly in angular power spectra:

$C_\ell^{\kappa\kappa} = \int [W^2(\chi)/\chi^2]\ P(k = \ell/\chi, z)\, d\chi$.

Rescaled spectrum $D_\ell^{\kappa\kappa} \equiv \frac{\ell(\ell+1)}{2\pi} C_\ell^{\kappa\kappa}$

In CAMB and CLASS, transfer functions govern physical evolution, while kernels govern geometric and observational projection into angular space.

Physically, what each "parameter" represents

ℓ Angular scale on the sky (small ℓ = large angles, large ℓ = small angles)

$C_\ell^{\kappa\kappa}$ → variance of projected mass fluctuations

$D_\ell^{\kappa\kappa}$ → variance per logarithmic interval in ℓ

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