Matter Power Spectrum Data

Custom high-resolution plot log P(k) vs.log k using a standard Linear-Theory Transfer-Function (LTTF) (e.g., from the CLASS or CAMB code) for ΛCDM-like cosmology. Y-axis P(k) normalized so that P(k = 0.2h/Mpc) = 1000. Shape is computed using a BBKS CDM transfer function with Planck-like parameters: **Ω*m* ≈ 0.315, *h* ≈ 0.67, ns ≈ 0.965Ω**

$Pk_{LTTF0} := READPRN(\text{"matter power spectrum table.txt"})$ $\quad Pk_{LTTF} := 1000 \cdot Pk_{LTTF0}^{\langle 1 \rangle}$

Consistent with Power Law Approximation: $\quad Pk_{Power_n} := 35\left(k_n\right)^{-2.45}$ $\quad k_{LTTF} := Pk_{LTTF0}^{\langle 0 \rangle}$

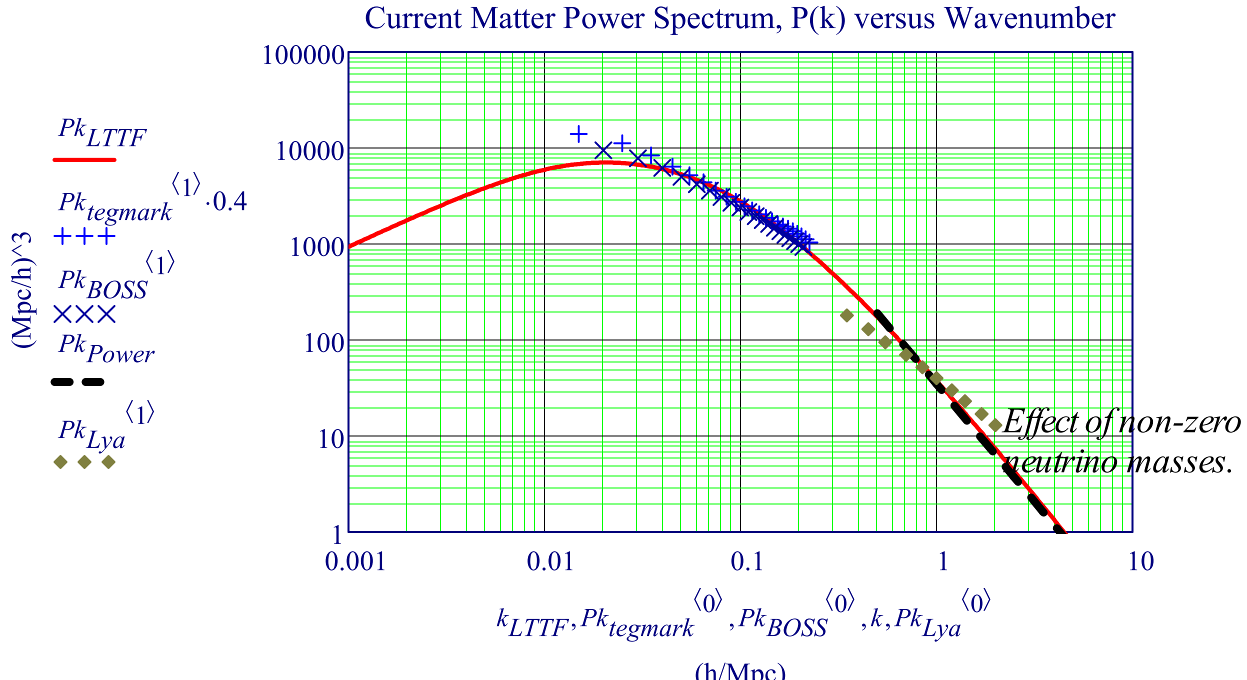
Matter Power Spectrum Data from Tegmark et al. 2003, BOSS

Tegmark 22 band-power measurements k from 0.02 to 0.3 h/Mpc
Galaxy clustering from SDSS main sample

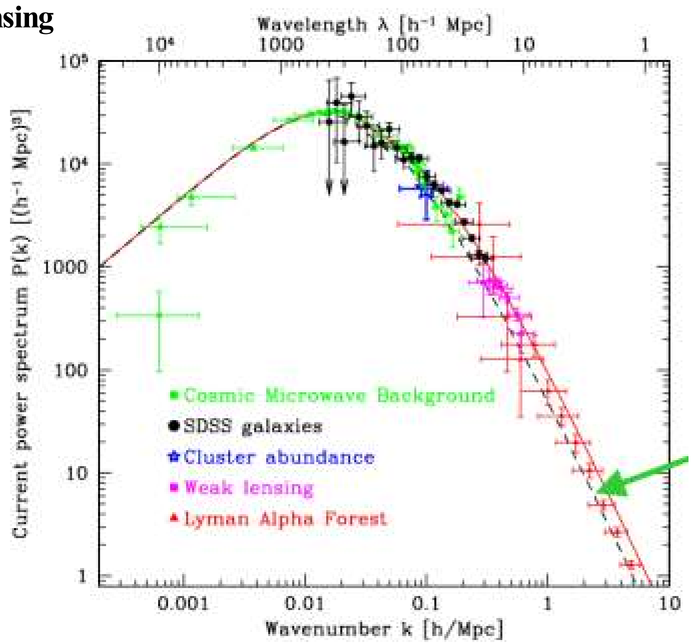
Pk_{Lya} Data: *The Atacama Cosmology Telescope, ACT: A Measurement of The Primordial Power Spectrum*
See Next Page for the Matter Distribution

$Pk_{tegmark} := READPRN(\text{"SDSS\_Tegmark2003\_Pk\_table.txt"})$ $\quad Pk_{Lya} := READPRN(\text{"Lya\_digitized\_high\_k.txt"})$

$Pk_{BOSS} := READPRN(\text{"BOSS\_DR12\_excerpt\_Pk\_table.txt"})$



Data from From CMB, LSS, Lyα, cluster abundances and weak lensing



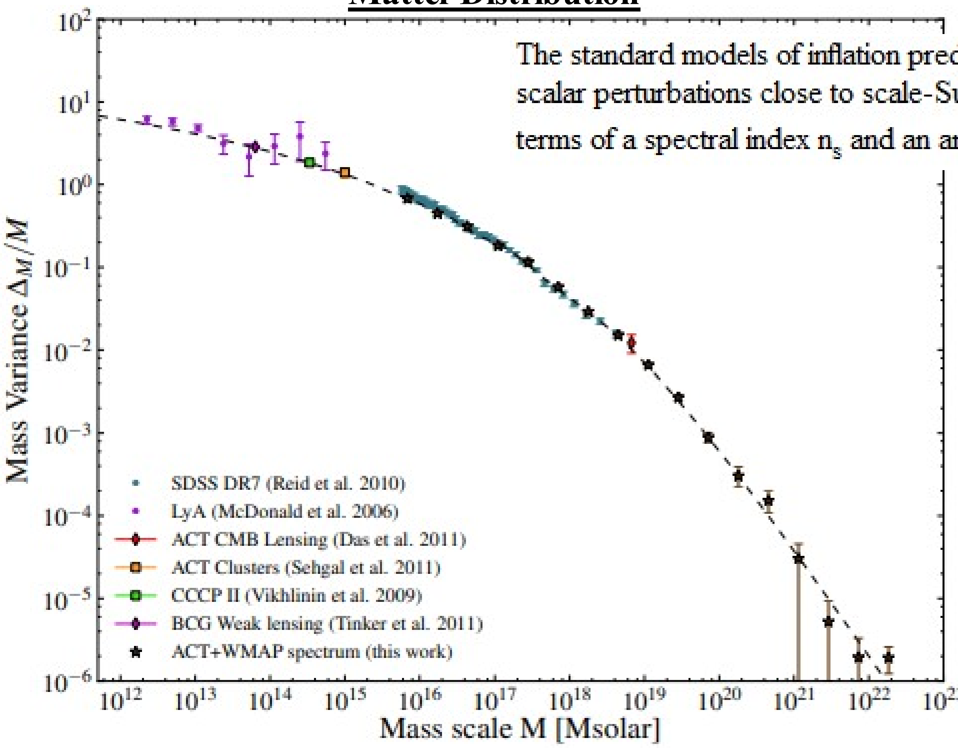
Effect of non-zero neutrino masses.

A Measurement of The Primordial Power Spectrum - Data from Atacama (ACT)

The Atacama Cosmology Telescope, ACT: A Measurement of The Primordial Power Spectrum, arXiv:1105.4887v1, Renee Hlozek, Joanna Dunkley1,2,3, Graeme Addison, John William Appe, October 10, 2018

Below is the primordial power spectrum of adiabatic fluctuations using data from the 2008 Southern Survey of the ACT. The angular resolution of ACT provides sensitivity to scales beyond $\ell = 1000$ for resolution of multiple peaks in the primordial temperature power spectrum, which enables us to probe the primordial power spectrum of adiabatic scalar perturbations with wavenumbers up to $k \approx 0.2\ \mathrm{Mpc}^{-1}$. We find **no evidence for deviation from power-law fluctuations** over two decades in scale. Matter fluctuations inferred from the primordial temperature power spectrum evolve over cosmic time and can be **used to predict the Matter Power Spectrum at late time**s; we illustrate the overlap of the matter power inferred from CMB measurements (which probe the power spectrum in the linear regime) with existing probes of galaxy clustering, cluster abundances and weak lensing constraints on the primordial power. This highlights the range of scales probed by current measurements of the matter power spectrum.

Matter Distribution



The standard models of inflation predict a power spectrum of adiabatic scalar perturbations close to scale-Such models are often described in terms of a spectral index n_s and an amplitude of perturbations Δ^2_R

$$\mathcal{P}(k) = \Delta^2_{\mathcal{R}} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

Transforming from units Power Spectrum to Mass Variance

$\Delta_M/M = [P(k)k^3/(2\pi^2)]^{1/2}$, allows one to visualize directly the **relationship between mass scale and variance**. While $\Delta_M/M \approx 1$ for $10^6\ M_\odot$ galaxies, the variance decreases as the mass increases and we probe the largest scales, covering ten orders of magnitude in the range of masses of the corresponding probes.

The reconstructed matter power spectrum: the stars show the power spectrum from combining ACT and WMAP data (top panel). The solid and dashed lines show the nonlinear and linear power spectra respectively from the best-fit ACT ΛCDM model with spectral index of $n_s = 0.96$ computed using CAMB and HALOFIT (Smith et al. 2003). The data points between $0.02 < k < 0.19\ Mpc^{-1}$ show the SDSS DR7 LRG sample, and have been deconvolved from their window functions, with a bias factor of 1.18 applied to the data. This has been rescaled from the Reid et al. (2010) value of 1.3, as we are explicitly using the Hubble constant measurement from Riess et al. (2011) to make a change of units from h^{-1}Mpc to Mpc. The constraints from CMB lensing (Das et al. 2011), from cluster measurements from ACT, CCCP (Vikhlinin et al. 2009) and BCG halos, and the power spectrum constraints from measurements of the Lyman–α forest (McDonald et al. 2006) are indicated. The CCCP and BCG masses are converted to solar mass units by multiplying them by the best-fit value of the Hubble constant, $\boldsymbol{h = 0.738}$ from Riess et al. (2011). The above panel shows the same data plotted on axes where we relate the power spectrum to a mass variance, Δ_M/M, and illustrates how the range in wavenumber k (measured in Mpc^{-1}) corresponds to range in mass scale of over 10 orders of magnitude. Note that large masses correspond to large scales and hence small values of k. This highlights the consistency of power spectrum measurements by an array of cosmological probes over a large range of scales

From Primordial Fluctuations to the Matter Power Spectrum

In the ΛCDM cosmological framework, the large-scale distribution of matter arises from primordial curvature perturbations generated during inflation and subsequently processed by well-understood microphysical effects in the early universe. While inflation predicts a nearly scale-invariant primordial power spectrum, the evolution through radiation domination and the coupling of baryons to photons significantly modify the growth of density perturbations after horizon entry. These effects imprint a characteristic turnover and a series of baryon acoustic oscillations (BAO) in the late-time matter power spectrum.

This section develops the connection between the primordial scalar power spectrum and the observed clustering of galaxies by introducing the linear transfer function ***T(k)***, with particular emphasis on the analytic **Eisenstein–Hu** (1998) formulation. The resulting **Matter Power Spectrum** $P_m(k)=P(k)T^2(k)$ provides a direct theoretical bridge between early-universe physics and large-scale structure measurements from galaxy surveys such as SDSS and DESI, which exploit both the broadband shape and the BAO feature as precision cosmological probes.

ΛCDM model incorporates:

8 Factors: Ω_m, Ω_b, Ω_{cdm}, Ω_Λ, H_0, n_s, A_s, τ,

- $\mathbf{A_s}$ Primordial amplitude
- n_s "Scalar Spectral Index
- Pivot Scale, k_0

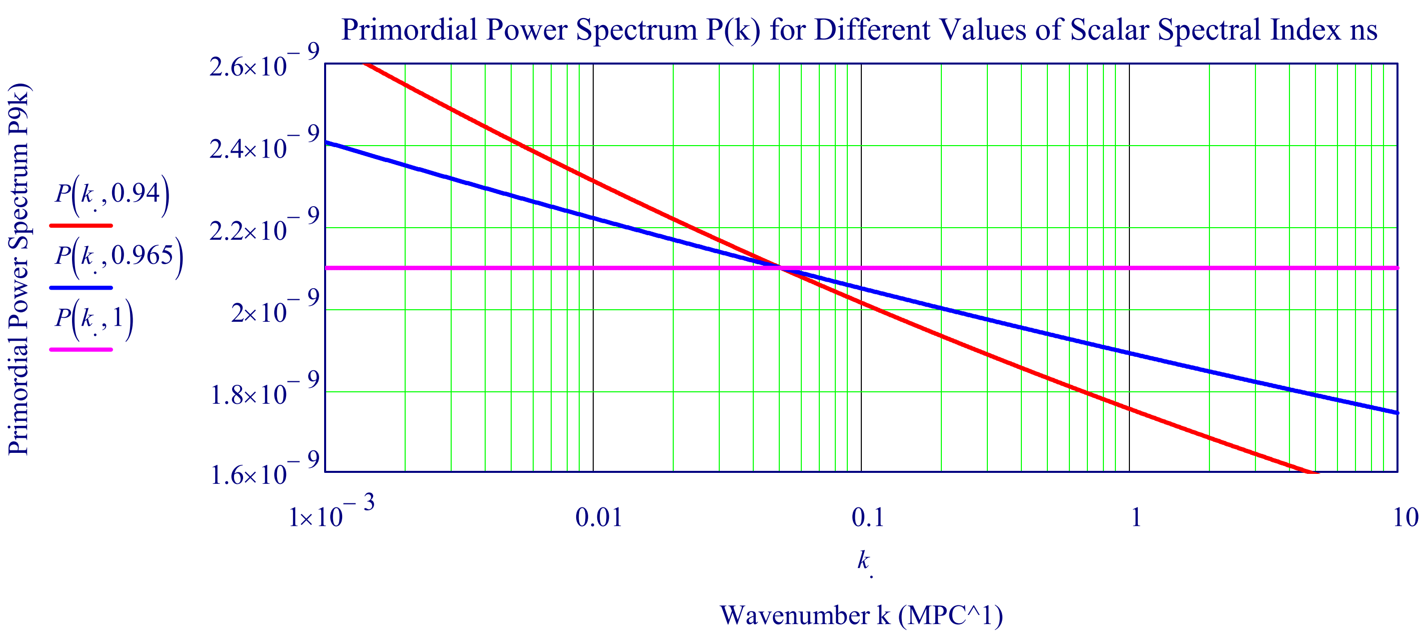
$$A_s := 2.1\cdot 10^{-9}$$

$$n_s := 1.0$$

$$k_0 := 0.05\cdot Mpc^{-1} \qquad P(k, n_s) := A_s\cdot\left(\frac{k\cdot Mpc^{-1}}{k_0}\right)^{(n_s-1)}$$

$n_s = 1$: Scale-invarianat (Harrison-Zeldovich) Spectrum

The farther n_s is below 1, the steeper the downard "**Tilt**".



The primordial scalar power spectrum is modified by causal microphysical processes after horizon entry. These effects are encoded in the **transfer function *T(k)***

Late-time matter spectrum:

$$P_m(k) = P(k)\cdot T^2(k)$$

While inflation predicts a nearly scale-invariant spectrum, **radiation domination and baryon coupling suppress growth on small scales**, producing the characteristic turnover observed in galaxy surveys.

At early times z > 3400, the universe was radiation-dominated. Small-scale density perturbations enter the horizon during radiation domination, where radiation pressure and rapid expansion inhibit gravitational growth, while baryons remain coupled to photons until recombination, delaying their collapse and suppressing power on small scales.

Radiation domination: trying to form galaxies in a hurricane.

Eisenstein–Hu (1998) Transfer Function with BAO

The **linear matter power spectrum** is

$$\boxed{P_m(k,z) = A_s \left(\frac{k}{k_0}\right)^{n_s-1} T^2(k)\, D^2(z)}$$

where the **total transfer function** is

$$\boxed{T(k) = f_c\, T_c(k) + f_b\, T_b(k), \qquad f_b = \frac{\Omega_b}{\Omega_m},\ f_c = 1 - f_b}$$

All quantities below follow Eisenstein & Hu (1998).

1. Background Quantities
Define physical densities:

$$\omega_m = \Omega_m h^2, \omega_b = \Omega_b h^2$$

CMB temperature factor:

$$\theta = \frac{T_{\mathrm{CMB}}}{2.7}$$

2. Matter–Radiation Equality Scale

$$\boxed{k_{\mathrm{eq}} = 0.0746\, \frac{\omega_m}{\theta^2}\ \mathrm{Mpc}^{-1}}$$

Define the dimensionless variable:

$$q = \frac{k}{13.41\, k_{\mathrm{eq}}}$$

3. Drag Epoch and Sound Horizon
Drag redshift z_d

$$z_d = \frac{1291\, \omega_m^{0.251}}{1 + 0.659\, \omega_m^{0.828}} \left[1 + b_1\, \omega_b^{b_2}\right]$$

where

$$b_1 = 0.313\, \omega_m^{-0.419} (1 + 0.607\, \omega_m^{0.674})$$
$$b_2 = 0.238\, \omega_m^{0.223}$$

Sound horizon at drag epoch

$$\boxed{s = 44.5\, \frac{\ln(9.83/\omega_m)}{\sqrt{1 + 10\, \omega_b^{0.75}}}\ \mathrm{Mpc}}$$

4. CDM Suppression Parameters

$$\alpha_c = a_1^{-f_b}\, a_2^{-f_b^3}$$

with

$$a_1 = (46.9\, \omega_m)^{0.670} [1 + (32.1\, \omega_m)^{-0.532}]$$
$$a_2 = (12.0\, \omega_m)^{0.424} [1 + (45.0\, \omega_m)^{-0.582}]$$

and

$$\beta_c = [1 + 0.944\, f_b]^{-1}$$

The Eisenstein–Hu (1998) transfer function

Provides an analytic description of the linear evolution of density perturbations in a baryon–cold dark matter universe, accurately capturing both radiation-era suppression and baryon acoustic oscillations. By modeling the imprint of the photon–baryon sound horizon and Silk damping, it reproduces the essential features of the linear matter power spectrum and serves as a widely used **analytic framework for BAO analyses in galaxy surveys.**

5. Core CDM Shape Function

$$\boxed{T_0(k;\alpha,\beta) = \frac{\ln\quad(e+1.8\,\beta\,q)}{\ln\quad(e+1.8\,\beta\,q) + \left(\frac{14.2}{\alpha} + \frac{731}{1+62.5\,q}\right)q^2}}$$

6. CDM Transfer Function $T_c(k)$

$$\boxed{T_c(k) = f\,T_0(k;1,\beta_c) + (1-f)\,T_0(k;\alpha_c,\beta_c)}$$

where

$$f = \frac{1}{1+(k\,s/5.4)^4}$$

This interpolates between large-scale and small-scale CDM growth.

7. Silk Damping Scale

$$\boxed{k_{\text{Silk}} = 1.6\,\omega_b^{0.52}\,\omega_m^{0.73}[1+(10.4\,\omega_m)^{-0.95}]}$$

8. Baryon Transfer Function $T_b(k)$
First define the **effective sound horizon shift**:

$$\tilde{s} = \frac{s}{[1+(\beta_b/ks)^3]^{1/3}}$$

with

$$\beta_b = 0.5 + f_b + (3-2f_b)\sqrt{(17.2\,\omega_m)^2+1}$$

Then

$$\boxed{T_b(k) = \left[\frac{T_0(k;1{,}1)}{1+(k\,s/5.2)^2} + \frac{\alpha_b}{1+(\beta_b/ks)^3}\right]\frac{\sin\,(k\tilde{s})}{k\tilde{s}}\;e^{-(k/k_{\text{Silk}})^{1.4}}}$$

where

$$\alpha_b = 2.07\,k_{\text{eq}}\,s\,(1+R_d)^{-3/4}$$

and

$$R_d = 31.5\,\omega_b\,\theta^{-4}\left(\frac{1000}{z_d}\right)$$

9. Final Result

$$\boxed{T(k) = f_c\,T_c(k) + f_b\,T_b(k)} \Rightarrow \boxed{P_m(k) = P(k)\,T^2(k)}$$

Explicit Connection to SDSS & DESI
What surveys observe
Galaxy surveys measure:

$$P_g(k,z) = b^2(z)\,P_m(k,z)$$

What Eisenstein–Hu provides

- **Broadband shape** → turnover from radiation domination
- **BAO oscillations** → from sin $(k\tilde{s})/(k\tilde{s})$
- **Silk damping** → exponential suppression at high k

Observational consequences

- **SDSS (BOSS/eBOSS)**
  Uses EH98 “wiggle” and “no-wiggle” templates to isolate BAO
- **DESI**
  Measures BAO with higher precision across multiple redshifts, constraining $D_A(z)$, $H(z)$, and Ω_m

PLOT SUMMARY

In ΛCDM, the Eisenstein–Hu (EH) transfer function encodes radiation-era suppression and baryon acoustic physics, producing both the turnover and the BAO oscillations observed in galaxy surveys such as SDSS and DESI.

Three EH98 Schematic Plots

These plots were made Using the Python Programs:

- Python-CAMB cross-validation (EH98 vs CAMB).py
- EH98_BAO_wiggle_program.py

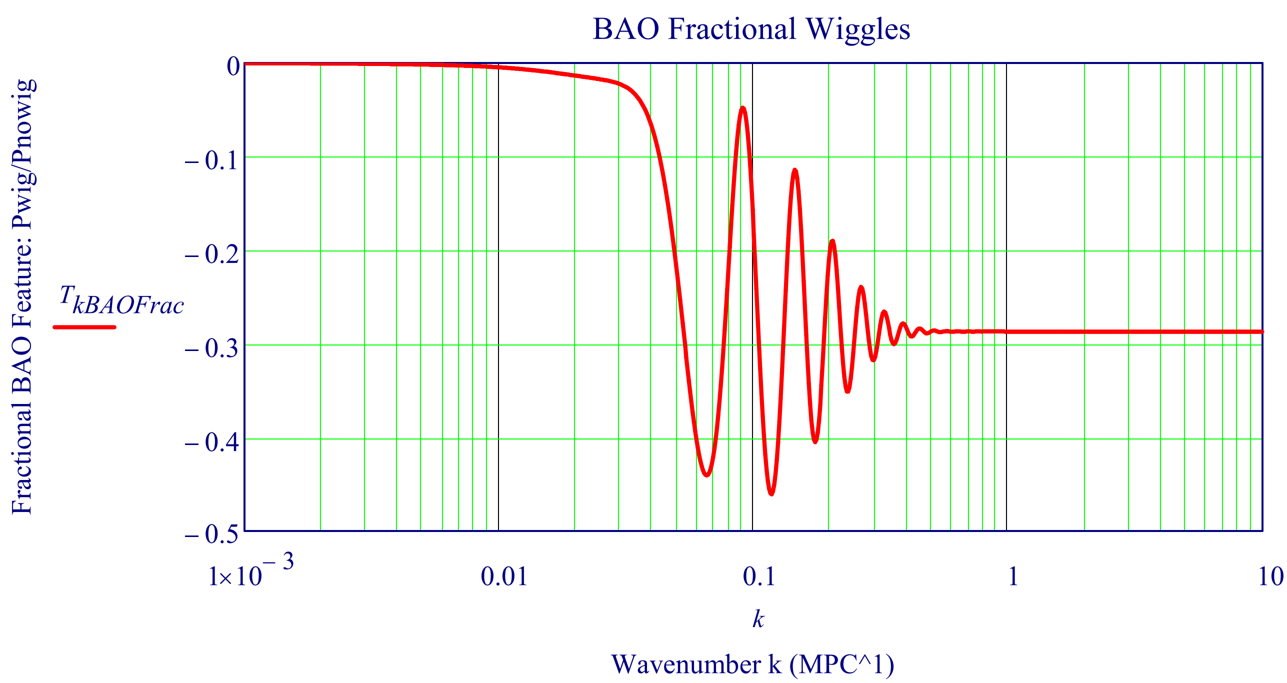
These Programs are available at: VXPhysics.com/Python

$$T_{wiggle} := READPRN(\text{"EH98\_BAO fractional wiggles.csv"})$$

$$k := T_{wiggle}^{\langle 0 \rangle} \qquad T_{kBAOFrac} := T_{wiggle}^{\langle 1 \rangle}$$

Fractional BAO Feature This plot isolates the baryon acoustic oscillation signal by dividing the full matter power spectrum by a smooth, no-wiggle reference spectrum. **The resulting fractional oscillation**s reveal the characteristic BAO scale that serves as a robust standard ruler in galaxy surveys such as SDSS and DESI.

$$P_{\text{wig}}/P_{\text{nw}} - 1$$



What this confirms (important)

- The plot you now see is exactly the intended Eisenstein–Hu BAO fractional feature:

$$\frac{P_{\text{wig}}(k)}{P_{\text{nw}}(k)} \to 1 \quad \Rightarrow \quad \frac{P_{\text{wig}}}{P_{\text{nw}}} - 1 \to 0$$

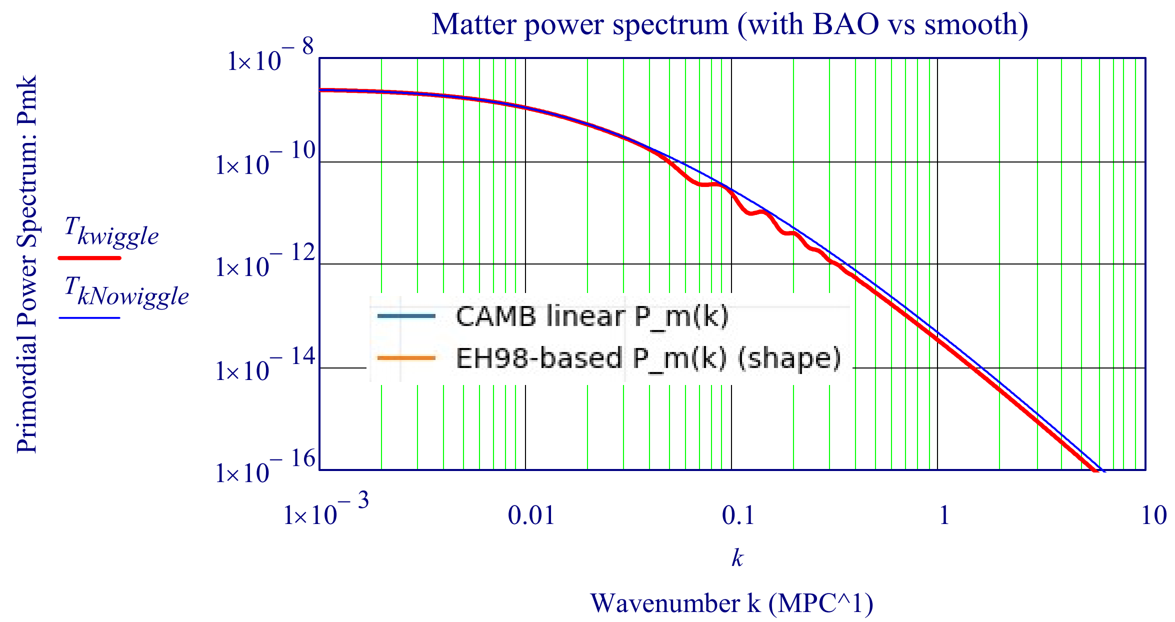
- The oscillations are present at the correct k-scales (around k ~ 0.05-0.3 h Mpc^{-1}
- The amplitude is a few percent, as expected for linear BAO physics
- This is pure EH98 analytic physics, not CAMB, not data, not non-lear

Linear Matter Power Spectrum $P_m(k)$

The linear matter power spectrum $P_m(k) = P_{\text{prim}}(k)T^2(k)$ rates how primordial inflationary fluctuations are processed by early-universe physics, producing a characteristic turnover and BAO oscillations at intermediate scales. Smooth curve highlights the broadband shape, while the oscillatory deviations trace the baryonic imprint on large-scale structure.

$T_{wiggle} := READPRN(\text{"EH98\_matter\_power\_wiggle\_nowiggle.csv"})$

$k := T_{wiggle}^{\langle 0 \rangle} \qquad T_{kwiggle} := T_{wiggle}^{\langle 1 \rangle} \qquad T_{kNowiggle} := T_{wiggle}^{\langle 2 \rangle}$



Transfer Function *T(k)*: Wiggle vs No-Wiggle

This plot shows **the Eisenstein–Hu linear transfer function *T(k)***, comparing the smooth suppression of power from radiation domination with the oscillatory modulation produced by baryon acoustic oscillations. The BAO features arise from sound waves in the photon–baryon fluid prior to recombination and are damped on small scales by photon diffusion.

$T_{tran\_fn} := READPRN(\text{"EH98\_transfer\_functions.csv"})$

$k := T_{tran\_fn}^{\langle 0 \rangle} \qquad T_{kBAO} := T_{tran\_fn}^{\langle 1 \rangle} \qquad T_{kNoWig} := T_{tran\_fn}^{\langle 2 \rangle}$