XXVIIA. The Discovery of the Accelerating Universe (2011)

What is the minimum redshift needed to determine if expansion of the universe is accelerating?

There isn't a single magic redshift, because it depends on (a) what you're measuring (SNe, BAO, cosmic chronometers), and (b) how strong your systematics are. But there is a clean physics answer:
What redshift scale contains "acceleration information"?
At very low redshift, the distance–redshift relation can be expanded as

$$d_L(z) \approx \frac{cz}{H_0}\left[1 + \frac{1-q_0}{2}z + \cdots\right]$$

The acceleration/deceleration parameter today, q_0, first appears at order z^2.

That means you need redshift large enough that the z^2 term is measurable, not buried in noise.

Minimum redshift in principle $z \gtrsim 0.1$ is the rough minimum where the curvature term $\propto z^2$ can start to become detectable in principle.

Minimum redshift in practice (what actually works)

- $z \gtrsim 0.3$ is a good practical minimum to get a statistically meaningful handle on acceleration from a Hubble diagram (e.g., Type Ia supernovae), because below that:
- peculiar velocities,
- local bulk flows,
- calibration systematics can dominate.

Best leverage for a clean "accelerating vs not" conclusion
$z \sim 0.5$ to 1 gives strong leverage, because the deviation between an accelerating and non-accelerating expansion history becomes much larger in the distance–redshift curve.

A useful rule of thumb
If the dataset only goes to only goes to $z \lesssim 0.2$, it's very hard to claim acceleration robustly.
If it reaches $z \gtrsim 0.5$, it's in the regime where the "curvature" of the Hubble diagram is clearly informative.

Saul Perlmutter (with the Supernova Cosmology Project, SCP) used Type Ia supernovae at redshifts extending to about $z \approx 0.8$, with the key acceleration signal coming from supernovae around $z \sim 0.4 - 0.7$.

What Perlmutter actually used (1998–1999)
📌 **Probe**
Type Ia supernovae as standardizable candles

📌 **Redshift range**
Low- z anxchor: $z < 0.1$ (nearby SNe for calibration)
High-z sample: up to $z \sim 0.83$
Critical leverage: $z \gtrsim 0.4$

Perlmutter's discovery of cosmic acceleration relied on Type Ia supernovae extending to redshift

$z \sim 0.5$ to 0.8,

where the curvature of the luminosity–distance relation becomes sensitive to the deceleration parameter.

Distance Modulus vs. Redshift for Type Ia Supernovae from the Supernova Cosmology Project

Lawrence Berkeley National Laboratory Data:
The Supernova Cosmology Project, SCP

Data from: https://supernova.lbl.gov/Union/figures/SCPUnion2.1_mu_vs_z.txt

$SCP := READPRN(\text{"SCPUnion mu vs z. Data Only.txt"})$ $\quad SCP := csort(SCP, 0)$

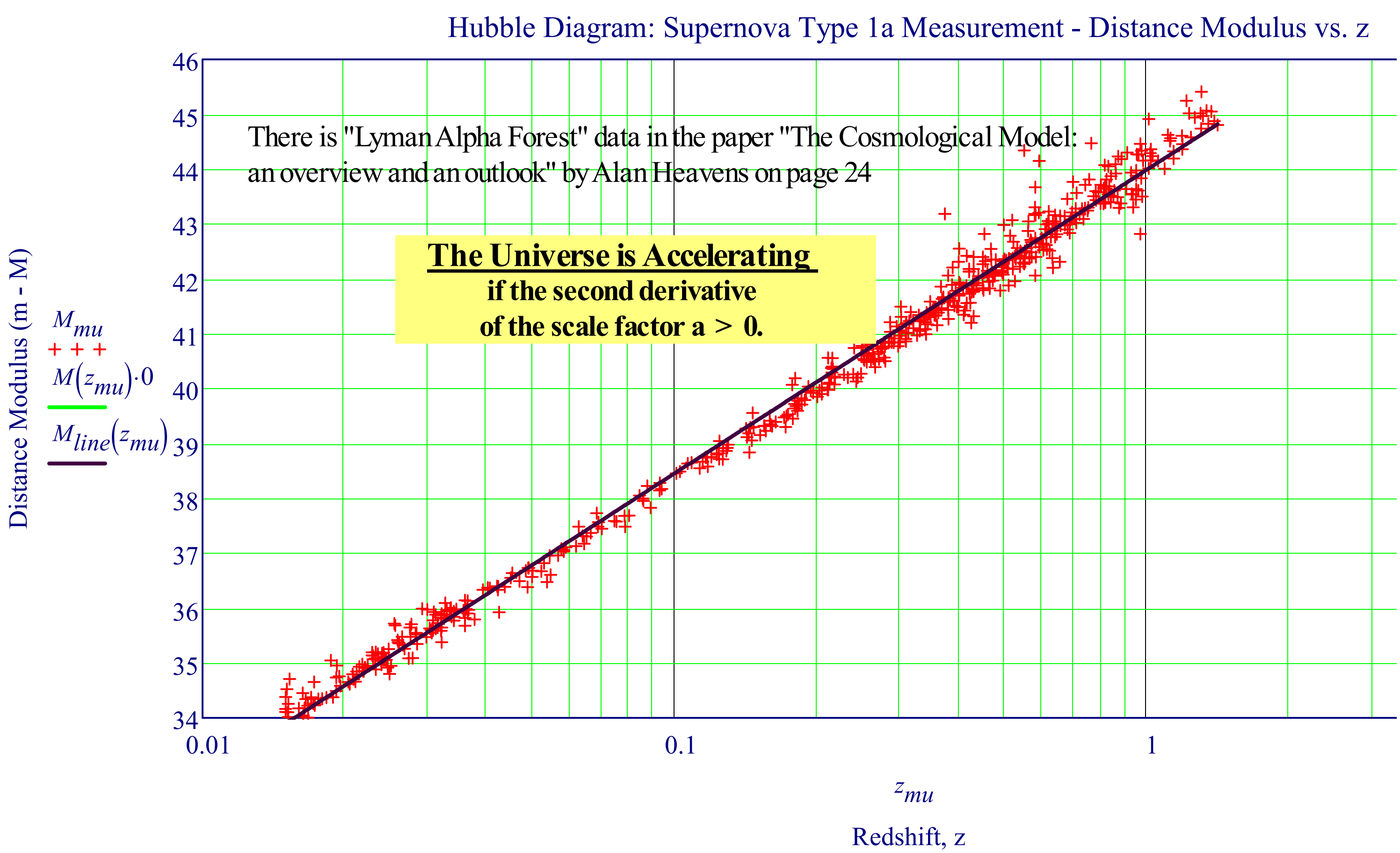
$z_{mu} := SCP^{\langle 0 \rangle}$ $\quad M_{mu} := SCP^{\langle 1 \rangle}$ $\quad vg := (1 \;\; 1 \;\; 1)^T$ $\quad rows(SCP) = 580$

Fit a Logfit Function and a Straight Line to Magnitude vs. Redshift Data

$ab := logfit(z_{mu}, M_{mu}, vg)$ $\quad M(z) := ab_0 \cdot ln(z + ab_1) + ab_2$

$ba := line(log(z_{mu}), M_{mu})$ $\quad M_{line}(z) := ba_0 + ba_1 \cdot log(z)$

$Diff(z) := M(z) - M_{line}(z)$



Find the Percent of z > 0.1 Supernovae that are above the Regression Line, Mline

$$PercentAbove := \left(\sum_{n=478}^{579} if\left(M_{mu_n} - M_{line}(z_{mu_n}) > 0, 1, 0 \right) \right) \frac{1}{100}$$

$PercentAbove = 64 \cdot \%$

- $\ddot{a} > 0 \rightarrow$ accelerating
- $\ddot{a} < 0 \rightarrow$ decelerating

This 64% shows that the Velocities of the High z Galaxies are statistically increasing faster than the Hubble Constant. **The Expansion is Accelerating. Visually, the second derivative of the scale factor, a > 0.**