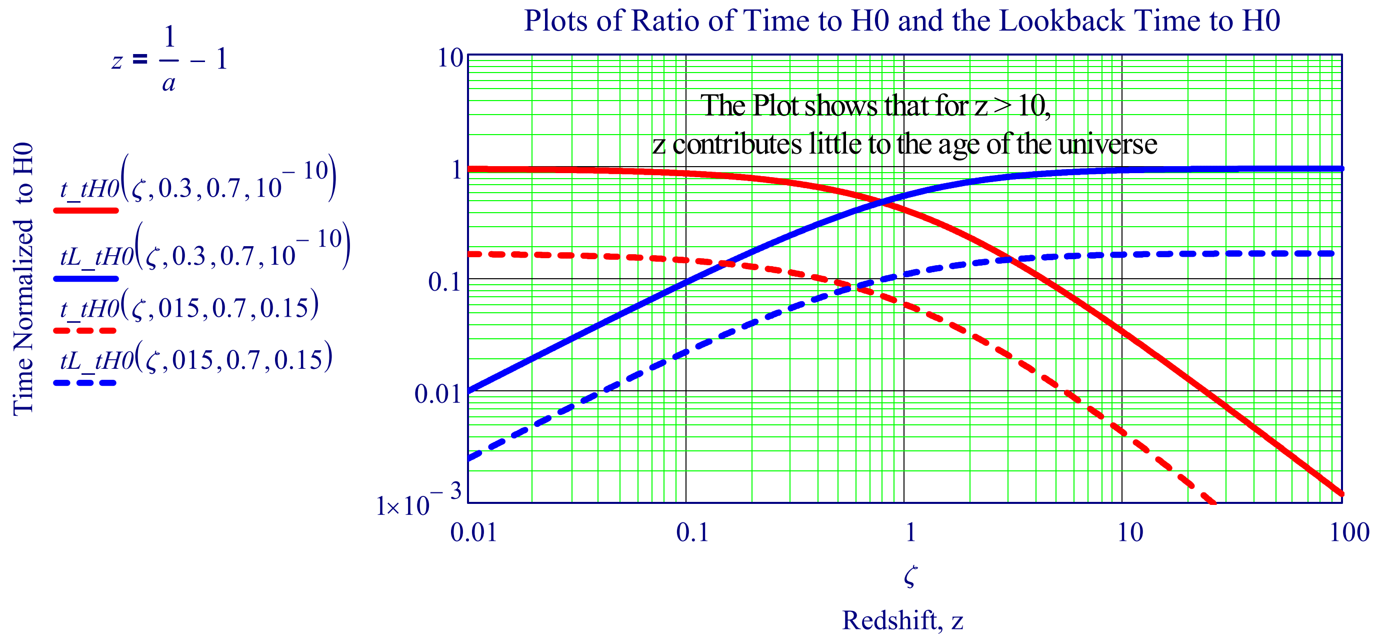
XXVIII. Lookback Time versus Red Shift and Reconstruction of High Z

$z = \frac{1}{a} - 1$



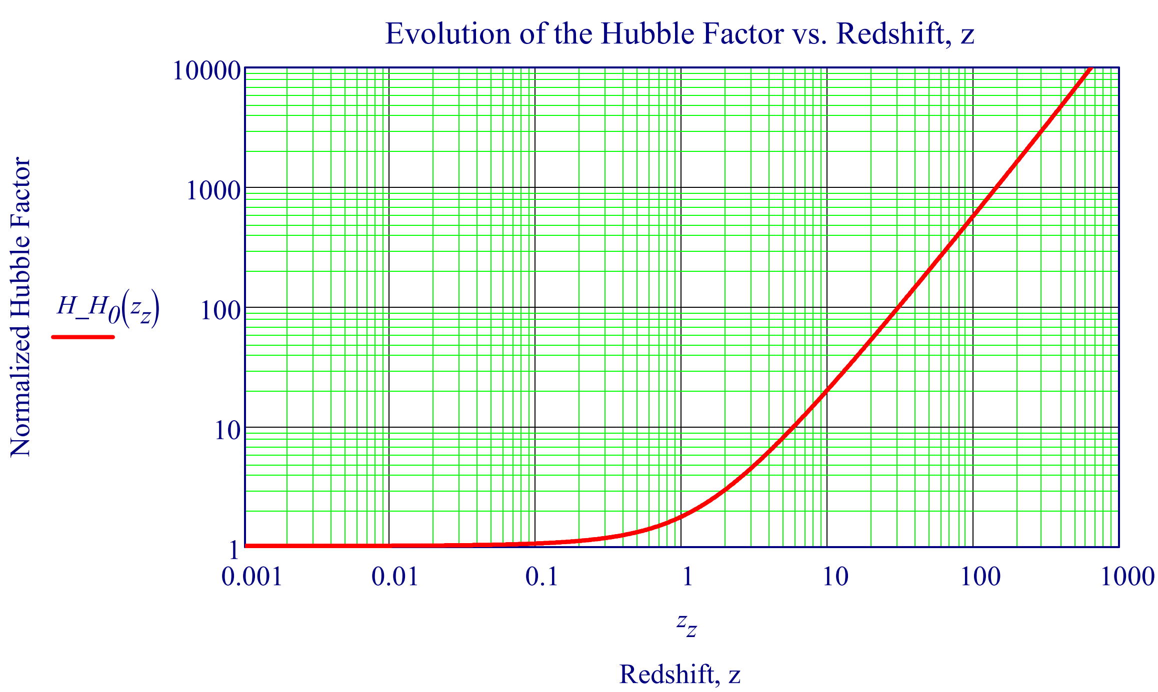
2023 Estimate z=10 is 13.30 Gyr

$t_{BB} := 13.8 Gyr \qquad t_{BB} \cdot tL\_tH0\left(10, 0.3, 0.7, 10^{-10}\right) = 12.844 \cdot Gyr$

Evolution of the Hubble Factor: Mass Conservation of non-relativistic matter implies $\rho_m \propto a^{-3} = (1+z)^3$. In the ΛCDM model, dark energy is assumed to behave like a cosmological constant: $\rho_\Lambda \propto a^0 = (1+z)^0$. The density of radiation (and massless neutrinos) scales as $\rho_r \propto a^{-4} = (1+z)^4$ because the number density of photons is $\propto a^{-3} = (1+z)^3$ and the mass $E/c^2 = h\nu/c^2$ of each photon scales as $E \propto \lambda^{-1} \propto (1+z)^1 \propto a^{-1}$.

Dynamical Equation Specifying the Evolution of the Hubble Factor of Our Universe

$\Omega_{r0} := 8.7 \times 10^{-5} \qquad \frac{H}{H_0} = H\_H_0(z) := \sqrt{\Omega_{m0} \cdot (1+z)^3 + \Omega_{\Lambda 0} + \Omega_{r0} \cdot (1+z)^4}$



Reconstruction of the Cosmic Equation of State for High Redshift (z = 2 to 5)

Refer to: A. M. Velasquez-Toribio, M. M. Machado & Julio C. Fabris, European Physical Journal, Vol 79, 2018

The accelerated expansion of the universe is one of the biggest problems of cosmology today. Among the different cosmological observables, **the cosmic equation of state (EoS) is of fundamental importance**, as it carries the kinematic and dynamic information of a given cosmological model. The reconstruction of these observables has been widely considered in the literature using different types of cosmological data, such as the following: Supernovae Ia, cosmic background radiation, clusters of galaxies, baryon acoustic oscillations (BAO), Hubble parameter data, $f\sigma_8$, and so on. Nevertheless, the reconstruction of this observable has **not been considered for high redshift**, in principle, **due to the lack of data for any redshift greater than 2.0**. However, this question is currently changing and we can consider **the reconstruction of the EoS (w(z)) for high redshifts**. Understanding in detail how *w(z)* evolves as a function of time is fundamental to know the nature of dark energy.

We use two methods to reconstruct the EoS: the first method makes use of distance measurements from Gamma-Ray Bursts (GRBs) & the second uses simulated data of the Hubble parameter generated by **Sandage–Loeb (SL) effect**. Sandage-Loeb (SL) test method directly measures the expansion history of the universe in the "z desert" of $2 < z < 5$. **In this paper we reconstruct *w(z)* using two model-independent approaches** the comoving distance is related to the luminosity distance by the relation: $D_L = (1+z)D_c$, and the comoving distance as a function of the Hubble parameter is defined by the following expression:

$$D_c = \frac{c}{H_0}\int_0^z \frac{dx}{h(x,\theta)} \qquad D_L = (1+z)D_c$$

where $h(z,\theta)$ is the dimensionless Hubble parameter, $H(z)/H_0$. In our case, it is given explicitly by Friedmann Equation

$$h^2(z,\Omega_{m0},\Omega_k) = \left\{\Omega_{m0}(1+z)^3 + \Omega_k(1+z)^2 + (1-\Omega_{m0}-\Omega_k)\exp\left[3\int_0^z \frac{1+w(z')}{1+z'}dz'\right]\right\},$$

Function of *w(z)*

where Ω_{m0} and Ω_k represent the matter density parameter and curvature respectively. In this paper, we assume that $\Omega_k = 0$, which matches the results of the Planck satellite. To derive from the previous equation, an expression for EoS, is useful. The definition

$$D_c = \frac{1}{\sqrt{-\Omega_k}}\sin\left(\sqrt{-\Omega_k}\int_0^z dz' \frac{H_0}{H(z')}\right) \qquad D_L = (1+z)D_c$$

where we have included the term curvature. We can use this equation above together with the equation of h(z) to **derive the equation of state *w(z)* as a function of *Dc* and its derivatives. *DL(z)* can be inverted to Find *w(z)*:**

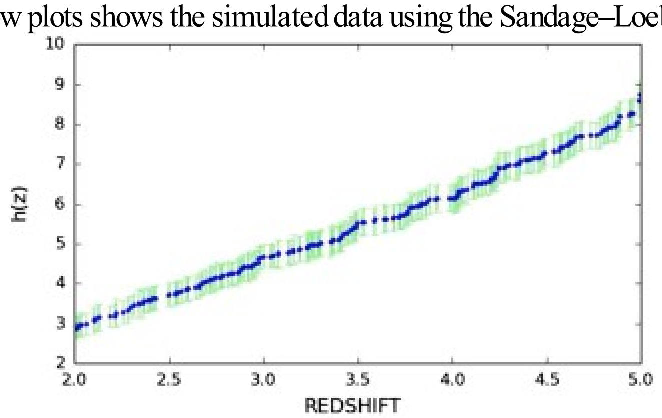
$$w(z) = \frac{2(1+z)(1+\Omega_k D_c^2)D_c'' - [(1+z)^2\Omega_k D_c'^2 + 2(1+z)\Omega_k D_c D_c' - 3(1+\Omega_k D_c^2)]D_c'}{3(1+z)^2[\Omega_k + (1+z)\Omega_m]D_c'^2 - (1+\Omega_k D_c^2)D_c'}$$

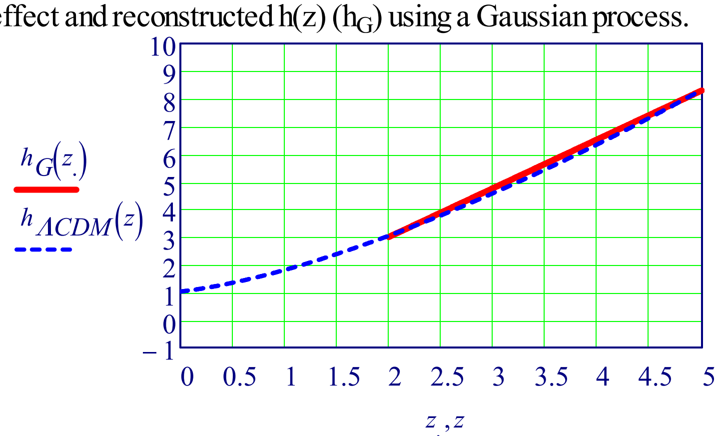
Derivation w(z) from DL
arxiv.org/abs/0807.4304v1
arxiv/astro-ph/0702670

where D_c' and D_c'' are the derivatives of D_c with respect to z. The ΛCDM Model for h, $h_{\Lambda CDM}(z)$ is:

$$h_{\Lambda CDM}(z) := \sqrt{\Omega_{m0}\cdot(1+z)^3 + \Omega_{r0}\cdot(1+z)^4 + \Omega_{\Lambda 0}} \qquad h_G(z) := 3 + \left[\frac{8.3-3}{3}\cdot(z-2)\right]$$

Below plots shows the simulated data using the Sandage–Loeb effect and reconstructed h(z) (h_G) using a Gaussian process.





N. Aghanim et al. [Planck Collaboration]. arXiv:1807.06209
P.A.R. Ade, et al. [Planck Collaboration], A&A 594, A13 (2016)

A. Sandage, Astrophys. J. 136, 319 (1962)
Parameter estimation with Sandage-Loeb test, arxiv1407.7123