Fine Tuning - The Flatness Problem Details and Calculations

Astronomy 275 Lecture Notes, Edward Wright

https://astro.ucla.edu/~wright/A275.pdf

Refer to: Section 6. Flatness

See Section VI for Energy Equation

Even in the general case with radiation, matter and vacuum densities, the energy equation is still

$$2E_{tot} = v^2 - \frac{8\pi G\rho R^2}{3} = H^2R^2 - \frac{8\pi G\rho R^2}{3} = \text{const}$$

For a Radiation-Dominated critical density Universe, $H_0 = 1/(2*t)$

$$\rho_{crit} = \frac{3\cdot H^2}{8\cdot \pi G} \qquad H = \frac{1}{2t} \qquad \rho_{crit}(t) := \frac{3}{32\pi G\cdot t^2} \qquad \frac{1}{H_0} = 4.344\times 10^{17}\,s$$

Then: $\Omega^{-1} - 1 = \dfrac{constant}{\rho\cdot a^2}$

Planck Time

$$t_{Pl} = \frac{\hbar}{m_{Pl}\cdot c^2} \qquad t_{Pl} := \sqrt{\hbar\cdot G\cdot c^{-\frac{5}{2}}} \qquad t_{Pl} = 13.495\,s\cdot 10^{-44}$$

Thus to find the value of

Density Parameter: $\mathbf{\Omega^{-1} - 1}$, we need to calculate the density, ρ, during the Planck era.

Planck Density

$$\rho_{Pl} := \frac{3}{32\cdot\pi\cdot G\cdot (t_{Pl})^2} \qquad \rho_{Pl} = 2.455\times 10^{94}\,\frac{kg}{m^3}$$

Stefan-Boltzmann Law: $P = \varepsilon\,\sigma\,A\,T^4$

For the density at the Planck time this is 2.16×10^{29} gm/cc if $g_{spin} = 106.75$.

Current Energy Density: $u = \dfrac{g_{spin}}{2}\cdot a\cdot T_0^{\,4}$

But the current value of $\boldsymbol{\rho R^2}$ is just $\rho_0 = \Omega_0\,\rho_{crit} = 1.8788\,\Omega_0\,h^2\times 10^{-29}$ gm/cc, Ω_0 is the density parameter at present.

We can calculate the temperature from: $\rho = \dfrac{a\cdot g}{2c^2}\cdot T^4$

To find the redshift from Entropy Density, s.

$$s = \frac{4}{3}\cdot\frac{g}{2}\cdot a\cdot T^4 = (1+z)^3\cdot s_o \qquad T_{pl} \approx 10^{32}\,\text{K}$$

The Current value of s is s_o $\quad s_o := 2890\cdot k_b\cdot\dfrac{J}{K}$

$$H^2 = \frac{8\pi G}{3}\left(\frac{\pi^2}{30}\right)g_*T^4$$

The quantity needed to find $\rho R^2 = \rho/(1+z)^2$ in order to find $\Omega - 1$ is given by equation 102 from Astronomy 275 Lecture Notes by Wright.

$$\Omega^{-1} - 1 = \frac{constant}{\rho\cdot a^2} \qquad \frac{\rho}{(1+z)^2} = \left(\frac{3.91}{g}\right)^{\frac{2}{3}}\cdot\left(\frac{a\cdot g\cdot T_o^{\,4}\cdot\rho}{2c^2}\right)^{\frac{1}{2}}$$

Calculate Ω required to **get Ω_0 (present density factor) between 0.95 and 1.05** with h = 0.71 and g = 106.75

Finally we get that **at the Planck time** Equation 103 from Astronomy 275:

$$\Omega^{-1} - 1 = \Omega_f(\Omega_0) := 8.7\cdot 10^{-59}\cdot\Omega_0\cdot 0.71^2\cdot\left(\Omega_0^{-1} - 1\right)$$

Then to get Ω_0 between 0.95 and 1.05 for Ω at Planck time

$$\Omega^{-1} - 1 = \Omega_f(0.95) = 0.219\cdot 10^{-59} \qquad \Omega^{-1} - 1 = \Omega_f(1.05) = -0.219\cdot 10^{-59}$$

Evaluating this in the Maple Programming Language to 62 digits gives:

Digits := 62;

$\Omega_{0.95}$:= 1 + evalf(0.219*10^(-59));

$\Omega_{0.95}$:= 1.0000000000000000000000000000000000000000000000000000000000022

$\Omega_{1.05}$:= 1 - evalf(0.219*10^(-59));

$\Omega_{1.05}$ = 0.99999999999999999999999999999999999999999999999999999999999781

Fine Tuning Analogy

The number 10^{-60} is, of course, very tiny. To make an analogy, in order to change the Sun's mass by one part in 10^{60}, you would have to add or subtract two electrons. Our very existence depends on the fanatically close balance between the actual density and the critical density in the early universe.

Thus to get Ω◦ between 0.95 and 1.05 with h = 0.71 and g = 106.75 requires 61 digits of Fine Tuning.

0.99999999999999999999999999999999999999999999999999999999999781 < Ω and

Ω < 1.0000000000000000000000000000000000000000000000000000000000022

The probability for Ω to randomly fall within this narrow window at the Planck Time is astronomically unlikely. Some unknown process must be at work for Ω to be so Finely Tuned.

2. The Cosmological Horizon Problem for the ΛCDM Theory

The universe appears to be homogeneous and isotropic on large scales. According to the COBE measurements,the cosmic background radiation (CBR) is uniform to a part in 10^4 on large scales (from about 10 " to 180 deg). Furthermore, the light element abundance measurements seem to indicate that the observable universe (bounded by the last scattering surface) was homogeneous by the time of nucleosynthesis. Hence, we would **expect the observable universe today (time t_0)** to have **been in causal contact by the time of nucleosynthesis t_n** ; otherwise the initial conditions of the universe would have to be extremely fine tuned in order for the causally disconnected patches to resemble one another as much as they do. However, in a Friedmann Robertson Walker (FRW) universe (a metric of $ds^2 = dt^2 - a(t)^2 dx^2$) that is matter or radiation dominated, upon naive extrapolation back to the singularity, one finds that there is a finite horizon length at the time of nucleosynthesis. Distant regions of space in opposite directions of the sky are so far apart that, assuming standard Big Bang expansion, they could never have been in causal contact with each other. This is because **the light travel time between them exceeds the age of the universe.**

Hence, for the observable universe to have been **in causal contact** by the time of nucleosynthesis, **the comoving horizon length** must have **been larger than the comoving distance** to the last scattering surface. In other words,

our observable universe today (when appropriately scaled back to the time of nucleosynthesis) **must have fit inside a causal region at the time of nucleosynthesis.**

The **comoving size L_o** of the **observable universe today** is

$$L_o = \int_{t_{dec}}^{t_0} \frac{dt}{a(t)}$$

where t_{dec} is the time of the **radiation decoupling** and t_0 is the time today (subscript 0 refers to today).

The **comoving size L_n** of the horizon at the **time of nucleosynthesis**

$$L_n = \int_0^{t_n} \frac{dt}{a(t)}$$

In order to explain causal contact of all points within our observable universe at the time of nucleosynthesis, we require $L_o < L_n$. However, this condition is not met in a naive FRW cosmology with matter or radiation domination. Even if we take t_n to be the the time of last scattering of CBR and not the nucleosynthesis time, we still have a horizon problem by a **factor of 10^5** . In **both matter or radiation domination cases**, the time dependence of the scalefactor is a power law with the index less than 1; in a **dust (matter) dominated universe**, $a \propto t^{2/3}$ and in a **radiation dominated** universe, $a \propto t^{1/2}$. Hence, in the naive FRW cosmology, $L_0 \approx t_0/a_0$ and $L_n \approx t_n /a(t_n)$, such that $L_o > L_n$ while **causal connection requires** $L_o < L_n$.

This is the horizon problem.

The above XXXII Inflation solves the horizon problem by having a period of accelerated expansion, with

$$\ddot{a} > 0$$

(a period of time when the universe was not dust or radiation dominated).

Can Geodesics in Extra Dimensions Solve the Cosmological Horizon Problem?

Daniel J. H. Chung 1, arXiv:hep-ph/9910235v2 12 Oct 2000

There is a possible non-inflationary solution to the cosmological horizon problem in scenarios in which our observable universe is confined to three spatial dimensions (a three-brane) embedded in a higher dimensional space. A signal traveling along an extra-dimensional null geodesic may leave our three-brane, travel into the extra dimensions, and subsequently return to a different place on our three-brane in a shorter time than the time a signal confined to our three-brane would take. Hence, these geodesics may connect distant points which would otherwise be “outside” the four dimensional horizon (points not in causal contact with one another).

2. The Horizon Problem (See Section V)

Consider matter-only universe:

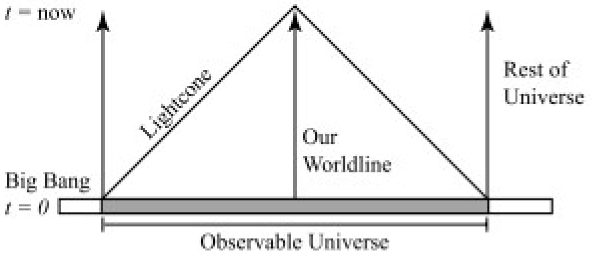
- Horizon distance $d_H(t) = 3ct$
- Scale factor $a(t) = (t/t_0)$ 2/3
- Therefore **horizon expands faster than the universe**, so new” objects are constantly coming into view

Consider CMBR:

- It decouples at $1+z \approx 1100$
- i.e., $t_{CMB} = t_0 / 10^{4.5}$
- Then $d_H\,(t_{CMB}) = 3ct_0 / 10^{4.5}$
- Now this has expanded by a factor of 1000 to $3ct_0 / 10^{1.5}$
- But horizon distance now is $3ct_0$
- So angle subtended on sky by one CMB horizon distance is only ≈ 2°

Horizons in the Universe

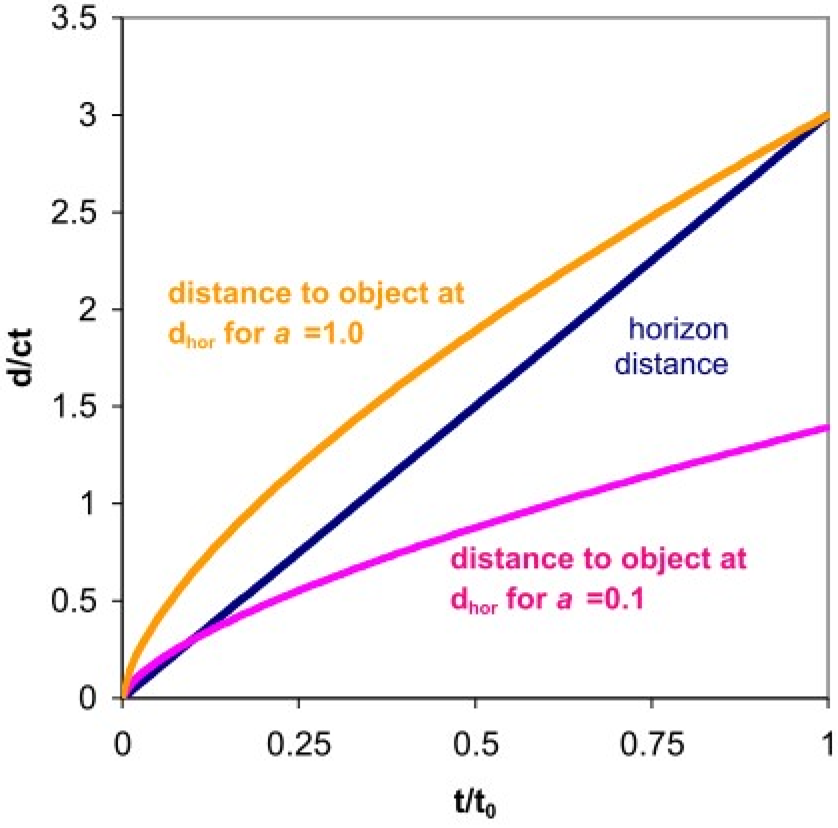
A space-time diagram illustrating the cosmic particle horizon, which defines the observable universe. If we trace our past lightcone back to the big bang, we find the most distant worldline that was ever within our past lightcone. The present distance to this worldline marks the particle horizon limit.



CMBR is Uniform to $\Delta T/T \approx 10^{-6}$

Yet the projected size of the particle horizon at the decoupling was ≈ 2° - these regions were causally disconnected - so how come?

=> Patches of CMB sky > 2° apart should not be causally connected!



Some Key Problems of the ΛCDM Cosmology - Continued:

3. Origin of Structure
4. Absence of Topological Defects
5. Low Entropy - The Second Law of Thermodynamics
6. Discrepancy Between Theoretically Estimated and Actual Value of Λ
7. The Hubble Tension
8 The Early Appearance of Supermassive Black Holes and Galaxies
9. Violations of Cosmological Principle: Isotropy, Homogeneity, and KBC Void