II D. ΛCDM Model Cosmological Eras for the Early Universe

To describe the conditions of the early universe quantitatively, recall the relationship between the average thermal energy of particle (E) in a system of interacting particles and equilibrium temperature (T) of that system where k_B and $\hbar$ are Boltzmann and Planck constants. Then we can calculate the Energy Values, E for the different eras.

$k_B := 1.380649 \cdot 10^{-23} \cdot \frac{J}{K}$ $\quad eV := 1.6 \cdot 10^{-19} C \cdot volt$ $\quad GeV := 10^{9} \cdot eV$ $\quad G := 6.67 \cdot 10^{-11} N \cdot m^{2} \cdot kg^{-2}$

$\hbar := 4.13 \cdot 10^{-15} eV \cdot s$ **Convert Energy to Temp (K):** $E(T) := k_B \cdot T$ **Temp (K) to Energy:** $T(E) := \frac{E}{k_B}$

Planck Era: Derived from Fundamental Constants Scale for Quantum Effects on Gravity. Create Mini Black Holes?

Planck Length

$$l_{pl} := \sqrt{\frac{\hbar \cdot G}{c^3}} \qquad l_{pl} = 4.045\, m \cdot 10^{-35}$$

Planck Time

$$t_{pl} := \frac{l_{pl}}{c} \qquad t_{pl} = 1.349\, s \cdot 10^{-43}$$

Planck Energy, Temp, Mass, Density

$$E_{pl} := \frac{\hbar}{2\pi \cdot t_{pl}} \qquad E_{pl} = 4.872 \times 10^{18} \cdot GeV$$

$$T(E_{pl}) = 5.646 \times 10^{31} K$$

$$M_{pl} := \sqrt{\frac{\hbar \cdot c}{G}} = 5.45 \times 10^{-8} kg$$

$$\rho_{pl} := \frac{M_{pl}}{\frac{4}{3} \cdot \pi \cdot l_{pl}^{3}} = 1.97 \times 10^{95} \frac{kg}{m^3}$$

GUT Era: $E_{GUT} \approx 10^{16}\ GeV$ $\quad T(10^{16} GeV) = 1 \times 10^{29} K$

See **XXIX. Early Universe Models: Quark-Gluon Plasma**

Nucleons: Form at energies ≈ rest mass of a proton, or 1 GeV.

$$T(1GeV) = 1 \times 10^{13} K$$

Atoms: Atoms form at an energy equal to the ionization energy of ground-state hydrogen (13 eV). The effective temperature for atom formation is therefore

$$T(13eV) = 1.507 \times 10^{5} K$$

Photons: The formation of atoms in the early universe makes these atoms less likely to interact with light. Therefore, photons that belong to the CMB must have separated from matter at a temperature T associated with 1 eV (the approximate ionization energy of an atom). The temperature of the universe at this point was

$$T(1eV) = 1.159 \times 10^{4} K$$

Recombination: Recombination is not instantaneous and photons keep re-ionizing hydrogen until the photon to baryon ratio drops enough.

The Saha equation describes the ionization equilibrium between electrons, protons, and neutral hydrogen:

- m_e is the electron mass $\quad m_e := 9.10938\, 10^{-31} kg$
- E_{ion} =13.6 V is the ionization energy of H.
- n_e is the electron density
- n_b is the baryon density, n_H Hydrogen density
- $n_b = n_e + n_H$ is the total baryon number density
  (protons + neutral hydrogen) and χ_e is their ratio.
- $n_\gamma(z)$ is the photon number density. @T=2.7K, n_γ ~410 cm^{-3}
- $\eta \sim 6 * 10^{-10}$ is the baryon to photon ratio.
- At recombination, the universe becomes
  ~50% ionized, so take $X_e \sim 0.5$
- $\xi(3)$ is the value of the Riemann zeta function at 3

$$\frac{n_e \cdot n_p}{n_H} = \left(\frac{2\pi \cdot m_e \cdot k_B \cdot T}{h^2}\right)^{\frac{3}{2}} \cdot e^{\frac{-E_{ion}}{k_B \cdot T}}$$

$$\frac{\chi_e^{2}}{1 - \chi_e} \cdot n_b = \left(\frac{2\pi \cdot m_e \cdot k_B \cdot T}{h^2}\right)^{2} \cdot e^{\frac{-13.6eV}{k_B \cdot T}}$$

$$n_b(T) := \left(\frac{2\pi \cdot m_e \cdot k_B \cdot T}{h^2}\right)^{\frac{3}{2}} \cdot e^{\frac{-13.6eV}{k_B \cdot T}}$$

$$\xi(n) := \sum_{n=1}^{\infty} \frac{1}{n^3} \qquad n_b(z) = \eta \cdot \frac{2\,\xi(3)}{\pi^2} \cdot \left(\frac{k_B \cdot T_{rec}}{h \cdot c}\right)^{3}$$

$$T_{recx} := \frac{13.6eV}{54 \cdot k_B} \qquad T_{recx} = 2919\, K$$

Putting it all together and solving for Trec gives:

$T_{rec} := 2970K$ $\qquad z_{rec} := 1100$

Reionization:

Reionization refers to a change in the intergalactic medium from neutral hydrogen to ions. The neutral hydrogen had been ions at an earlier stage in the history of the universe, thus the conversion back into ions is termed a reionization. The reionization was driven by energetic photons emitted by the first stars and galaxies. We will use a combination of observational data and cosmological modeling.

We ask the question: **What was the time or redshift for the Reionization Era**

Mega Parsec, Mpc:

See Section XXII: Optical Depth - Reionization Optical Depth Parameter, τ

$$Mpc := 3\cdot 10^{19}\,km$$

$$\tau = c\cdot\sigma_e\cdot\int_{t_{zcmb}}^{t_0} n_e(t)\,dt \qquad \tau = c\cdot\sigma_e\cdot\int_{t_{zcmb}}^{t_0} n_e(t)\cdot\left(\frac{d}{dz}t\right)dz \qquad \tau = \frac{2\cdot c\cdot\sigma_T\cdot n_{e0}}{3H_0}\cdot\sqrt{\Omega_m}\cdot\left[\left(1+z_{re}\right)^{\frac{3}{2}}-1\right]$$

- $n_e(z)$ is the free electron density,
- σ_T is the Thomson cross-section,
- dt/dz depends on the cosmology

$$H_0 := 67.4\frac{km}{s}\cdot Mpc^{-1} \qquad \Omega_m := 0.315 \qquad \tau := 0.054$$

$$\sigma_T := 6.652\cdot 10^{-29}\,m^2 \qquad n_{e0} := 2\cdot 10^{-7}\,cm^{-3}$$

Estimation Method for Redshift at Reionization

For a rough estimate using CMB optical depth τ, you can invert the equation assuming instantaneous reionization

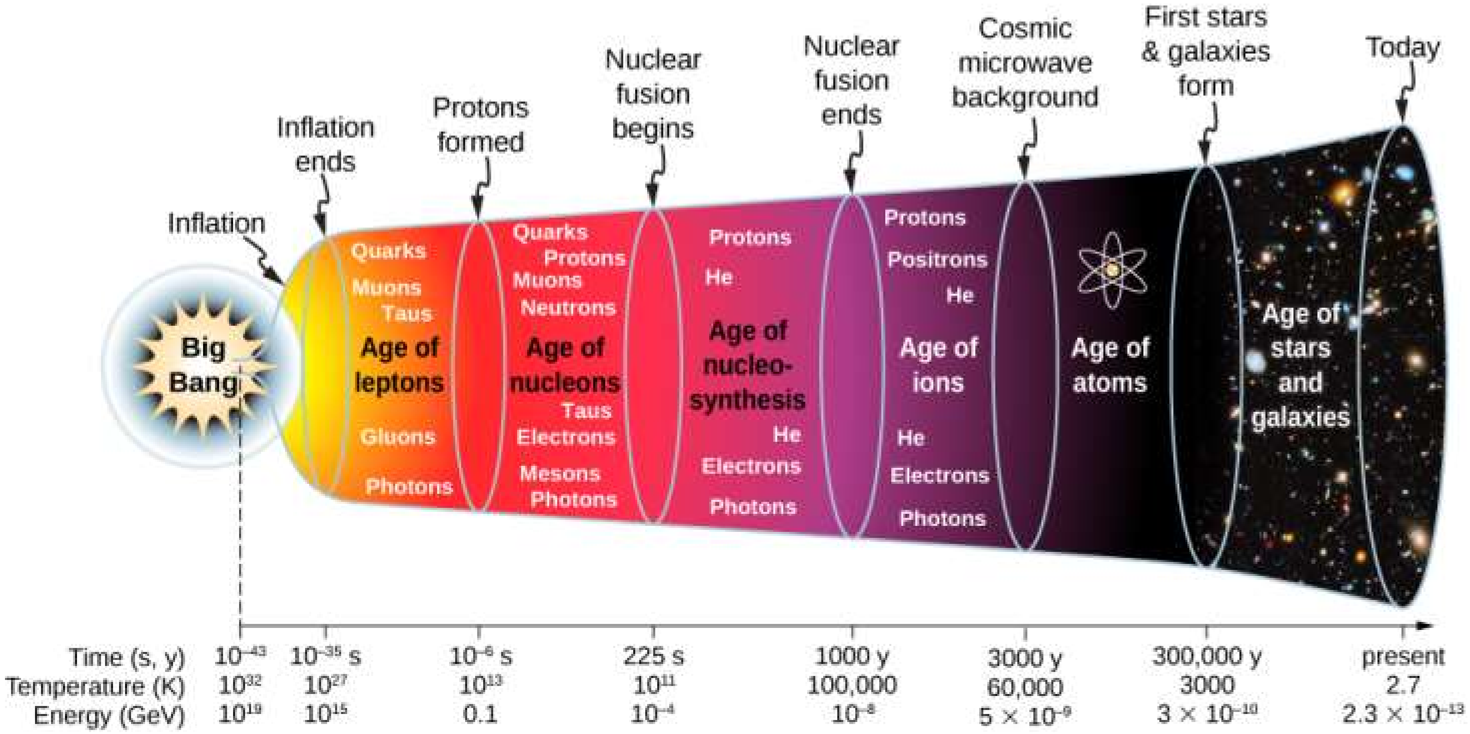
Solving for z_{re} you get

$$z_{re} := \left(\frac{3H_0\cdot\tau}{2\cdot c\cdot\sigma_T\cdot n_{e0}\cdot\sqrt{\Omega_m}}+1\right)^{0.66}-1 \qquad z_{re} := 7.7$$

Estimate the temperature of the universe at reionization, we use the fact that the temperature of the cosmic microwave background (CMB) scales with redshift as:

Given Current CMB Temperature: $T_{cmbr} := 2.725K$ $\qquad T_z(z) := T_{cmbr}\cdot(1+z)$ $\qquad T_z(7.7) = 23.707\,K$

CMB: $\mu eV := 10^{-6}\,eV$ $\qquad E\left(T_{cmbr}\right) = 235.142\cdot\mu eV$



Hypothetical and Observable Thermal Sequence For the ΛCDM Theory

https://universe-review.ca/F02-cosmicbg01.htm

The relics and observables are physical facts,

while the interpretations of the events are mostly theories or conjectures. **See XXXII for Plot of Time Evolution of Eras.**

At any given time, **temperature translates** to a **characteristic mass** of particles (**$kT \approx mc^2$**), which dominate that epoch.

Era	**Time @ end of era**	**Size (observable) @ end of era**	**Energy/Temp**	**Relics & Observables**	**Events (as re-constructed from theories)**
Planck era (???)	< 5.4x10^{-44} sec	< 1.6x10^{-33} cm	> 1.2x10^{19} Gev	(3+1)D space-time; Cosmic Expansion	Expansion started from a point to Planck scale; all forces united into one
GUT era	<10^{-35} sec	< 10^{-26} cm	> 10^{14} Gev > 10^{27} K	High energy cosmic rays; fundamental interactions	Separation of spacetime and matter; separation of gravitational, strong, and electroweak forces
Inflation (Rate of Expansion >>> c)	< 10^{-32} sec	< 30000 cm	10^{14} Gev	Un-observable universe; large scale structures	Reheating; Unstable vacuum; quantum fluctuations
Top Quark era Electro-weak era Quark-Gluon era QCD Domain	≈ 10^{-25} sec < 10^{-11} sec ≈ 10^{-10} sec ≈ 10 μsec	< 10^{14} cm	> 8 Tev > 300 Gev > 150 Gev > 200 Mev	Radiation; excess of matter over anti-matter; separation of force (bosons), and matter (fermions) fields	Radiation released in reheating; baryon asymmetry; separation of weak and E-M forces; origin of mass
Hadron era	< 1 sec	< 10^{20} cm	> 1.7 Mev	Formation of hadrons	Axion as dark matter
Weak decoupling	< 4 min	< 4x10^{20} cm	> 100 kev	neutron/proton ratio fixed	Neutrinos decouple
Nucleosynthesis	< 1/2 hour	< 10^{21} cm	> 40 Kev	Fraction of Light elements	Nuclear reactions freeze out, stable nuclei form
Radiation era **Matter era**	< 0.24 My	< 2x10^{27} cm	> 0.6 ev	Mass density fluctuations	Matter density finally exceeds radiation density
General Cosmology Time Era: Astronomical Observable, Relics, and Measureable					
Recombination $p^+ + e^- \rightarrow H + \gamma$	< 0.3 My	< 3x10^{27} cm	> 3000°K	CMBR 1965 Penzias and Wilson	e- and p+ recombine into H atoms, universe became transparent to light
----- Redshift----- *z = 1100 to 30* Dark Ages	< 1 Gy	< 2x10^{28} cm	> 100°K	21 cm radio emission, First stars, heavy elements	mass fluctuations grow, first small objects coalesce, reionization
Galaxy formation	< 2 Gy	< 2.5x10^{28} cm	> 70°K	Stars, quasars, galaxies	Collapse to galactic systems
Bright age of Galactic Clusters	< 12 Gy	< 4.5x10^{28} cm	> 3°K > 0.00025 ev	Solar system; decline of stellar formation from peak	dark energy became dominant; formation of clusters of galaxies
Present era	~ 13.7 Gy	~ 4.7x10^{28} cm	~ 2.73°K	Supercluster	Large scale gravitational instability