VII. Equations and Values of Constants for Cosmological Parameters:

Hubble & Scale Factors, z, Ωs, Density, Temp, V

Definitions and Equations below came from: *Introduction to Cosmology*, by Barbara Ryden[2]

Plots of these Cosmic Parameters are on the Following Pages

Define Constants

$c = 299792.458 \cdot \frac{km}{s}$ $G := 6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$ $H_0 := \frac{1}{4.355 \cdot 10^{17} s}$

Seconds per Billion (Giga) Years

$Gyr := 3600 \cdot 24 \cdot 365.24 \cdot 10^9 \cdot s$

$H_0 = 68.886 \cdot \frac{km}{s} \cdot (Mpc)^{-1}$

Create an Exponential Time: Order of Magnitude, OM, Scale Factor ai, Spanning 26 Orders of Magnitude:

$N26 := 10^{-26}$ $OM := 26$ $i := 0 .. 100 \cdot OM + 400$ $a_i := 10^{0.01 \cdot i - OM}$ $a_0 = 1 \cdot N26$ $a_{3000} = 10000$

Densities and Curvature of our Universe

Critical Density

In flat universe total density ρ = critical density ρ_0

$\rho_0 = \frac{3 \cdot H_0^2}{8 \cdot \pi \cdot G}$ $\rho_0 = 8.644 \frac{kg}{m^3} \cdot 10^{-27}$

Normalized radiation energy density for photons + neutrinos

$\Omega_{r0} := \frac{4.005 \cdot 10^{-14}}{\rho_0 \cdot c^2} \cdot (1 + 0.69) \cdot \frac{J}{m^3}$ $\Omega_{r0} = 0$

Dark matter + baryonic matter

$\Omega_{m0} := 0.268 + 0.049$ $\Omega_{m0} = 0.317$

Curvature Parameter

$\Omega_{\Lambda 0} := 1 - \Omega_{r0} - \Omega_{m0}$ $\Omega_{\Lambda 0} = 0.683$

DEFINE: Ω, H, da_dt, Proper time, t, Diameter, Velocity, Mass Ratios, H(z)

Inflation, i

Dark energy for flat universe

$H_i \approx t_{GUT}^{-1} \approx 10^{36}\ sec^{-1}$

Hubble Parameters

$H_i := H_0 \cdot \sqrt{\frac{\Omega_{r0}}{(a_i)^4} + \frac{\Omega_{m0}}{(a_i)^3} + \Omega_{\Lambda 0}}$

Scale Factor, a

$a(t) = \frac{1}{1 + z}$

Redshift,

$z = \frac{1}{a} - 1$ $a = \frac{1}{z + 1}$

Friedmann equation for a flat universe
after inflation ends
and radiation epoch begins

$$\dot{a} = H_\circ \left(\Omega_{k\circ} + \Omega_{v\circ} a^2 + \Omega_{m\circ}/a + \Omega_{r\circ}/a^2\right)^{1/2}$$

$$\frac{d}{dt} a = da\_dt(a) := H_0 \cdot \sqrt{\frac{\Omega_{r0}}{a^2} + \frac{\Omega_{m0}}{a} + \Omega_{\Lambda 0} \cdot a^2}$$

Calculate the Cosmic Proper Time (t) and Lookback time (tL). Inflation Epoch Ends at 10^- 33 seconds

Distance to a galaxy is defined as the **proper distance $d_p(t)$**. The length of time light has traveled t_0 - t_e is **lookback time**, t_L.

Inflation Era 10^{-35} to 10^{-33}

$$t_i := \int_0^{a_i} \frac{1}{da\_dt(a)}\,da \qquad t_{L_i} := \int_0^{z_i} \frac{1}{(1+z\varepsilon)\cdot H(z\varepsilon)}\,dz\varepsilon \qquad \frac{t_{100\cdot OM}}{Gyr} = 13.096 \qquad \frac{t_{3000}}{Gyr} = 165.792$$

$$X33 := 10^{-33} \qquad t_1 = 2.443\,s\cdot X33 \qquad t_{3000} = 165.792\cdot Gyr \qquad Now := t_{100\cdot OM}\cdot s^{-1}$$

Numerical Integration: Integral dD (a, b, n)

$$dD(a) := \frac{2\cdot c}{a\cdot da\_dt(a)}$$

$$Integral\_dD(a,b,n) := \begin{cases} \left[\frac{dD(a)+dD(b)}{2}\cdot(b-a)\right] & \text{if } n \le 1 \\ \text{otherwise} & \\ \quad h \leftarrow \frac{b-a}{n} \text{ if } n > 1 \\ \quad \frac{h}{2}\cdot\left[dD(a) + \left(2\cdot\sum_{i=1}^{n-1} dD(a+i\cdot h)\right) + dD(b)\right] \end{cases}$$

Calculate the Diameter, D, in Meters of Observable Universe Dou = 2*comoving distance

$$dD(a) := \frac{2\cdot c}{a\cdot da\_dt(a)} \qquad Initial := 1\cdot 10^{-100} \qquad da\_dt_i := da\_dt(a_i)$$

$$Dou_i := Integral\_dD(Initial, a_i, 500) \qquad Dou_0 = 279.757\,m \qquad \frac{Dou_{100\cdot OM}}{c\cdot Gyr\cdot s} = 88.602\,\frac{1}{s}$$

D = Scaled Up Diameter of Universe that was formerly observable at 10^{-33} second

$$D_i := \frac{a_i}{a_0}\cdot Dou_0 \qquad D_0 = 279.757\,m \qquad \frac{D_{100\cdot OM}}{Dou_{100\cdot OM}} = 33.375$$

$$Recombination\_Time := 3600\cdot 24\cdot 365.24\cdot 370000$$

Calculate Recessional Velocities

$$vrou_i := H_i\frac{Dou_i}{2} \qquad Vou := \frac{4\pi}{3}\cdot\left(\frac{Dou}{2}\right)^3$$

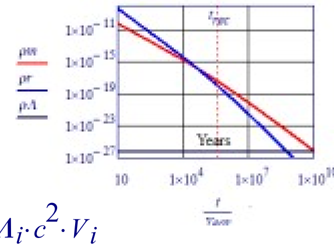
$$V_i := \left(\frac{a_i}{a_0}\right)^3\cdot Vou_0 \qquad vr_i := H_i\cdot\frac{D_i}{2}$$

Temperature (K)

$$Temp_i := \frac{2.725}{a_i}$$

Mass Densities: Radiation, Mass, Λ, and Total.   See Density (ρ) Plots in Section XXVIII

$$\rho r_i := \frac{\Omega_{r0}}{(a_i)^4}\rho_0 \qquad \rho m_i := \frac{\Omega_{m0}}{(a_i)^3}\rho_0 \qquad \rho\Lambda_i := \Omega_{\Lambda 0}\rho_0 \qquad \rho := \rho r + \rho m + \rho\Lambda$$



Mass (Mou) and Energy (Eou) of Dark and Baryonic Matter and Energy

$$Mou_i := \rho m_i \cdot Vou_i \qquad Erou_i := \rho r_i \cdot c^2 \cdot Vou_i \qquad Emou_i := \rho m_i \cdot c^2 \cdot Vou_i \qquad E\Lambda_i := \rho\Lambda_i \cdot c^2 \cdot V_i$$

$$M_{\rho v_i} := \rho m_i \cdot V_i \qquad Er_i := \rho r_i \cdot c^2 \cdot V_i \qquad Em_i := \rho m_i \cdot c^2 \cdot V_i \qquad E\Lambda ou_i := \rho\Lambda_i \cdot c^2 \cdot Vou_i$$

$$Eou := Erou + Emou + E\Lambda ou \qquad E := Er + Em + E\Lambda$$

Radiation - Matter Equality

$$arm := \frac{\rho r_{700}}{\rho m_{700}}$$

Matter - Lambda Equality

$$am\Lambda := \sqrt[3]{\frac{\rho m_{2600}}{\rho\Lambda_{2600}}} \qquad am\Lambda = 0.774$$

$$ar_i := \sqrt[4]{4 \cdot \Omega_{r0}}\left(H_0 \cdot t_i\right)^{\frac{1}{2}} \qquad am_i := \sqrt[3]{2.25 \cdot \Omega_{m0}}\left(H_0 \cdot t_i\right)^{\frac{2}{3}} \qquad a\Lambda_i := am\Lambda \cdot e^{\sqrt{1-\Omega_{m0}} \cdot H_0 \cdot t_i}$$

$$C_{inf} = 8\pi G \cdot \frac{f}{3} + \frac{\Lambda}{3} \qquad a_{inflation}(t) = e^{\sqrt{C_{inf}} \cdot t} \qquad One\_Year := 3600 \cdot 24 \cdot 365 \qquad Now := t_{100 \cdot OM} \cdot s^{-1}$$

Plots of the Ratio of Lookback time to H0 (tL tH0) and the Ratio of Time to H0, (t tH0)

$$tL\_tH0\left(z, \Omega_{0m}, \Omega_{0\Lambda}, \Omega_{0r}\right) := \int_0^z \frac{1}{(1+z\xi) \cdot \sqrt{\Omega_{0m} \cdot (1+z\xi)^3 + \Omega_{0\Lambda} + \Omega_{0r} \cdot (1+z\xi)^4}}\, dz\xi$$

$$tL\_tH0(1000, 0.3089, 0.6911, 0.001) = 0.952$$

$$t\_tH0\left(z, \Omega_{0m}, \Omega_{0\Lambda}, \Omega_{0r}\right) := \int_0^{(1+z)^{-1}} \frac{a}{\sqrt{\Omega_{0m} \cdot a + \Omega_{0\Lambda} \cdot a^4 + \Omega_{0r}}}\, da$$

$$t\_tH0(1000, 0.1, 0.7, 0.2) = 0$$

Comoving Distance

$$z = \frac{1}{a} - 1$$

$$D_{Cz}\left(z, \Omega_{0m}, \Omega_{0\Lambda}, \Omega_{0r}\right) := \int_{(1+z)^{-1}}^{1} \frac{1}{\sqrt{\Omega_{0m} \cdot a\xi + \Omega_{0\Lambda} \cdot a\xi^4 + \Omega_{0r}}}\, da\xi$$

Apparent Magnitude-Redshift Relation (Mukhanov) Eq 2.81 (See Section X of this Paper)

For Comoving Distance, χ_{em}

$$\chi = \int_{t_{em}}^{t_0} \frac{dt}{a(t)} \qquad \chi_{em}(z, \Omega_m) := \int_0^z \frac{1}{\sqrt{\Omega_m \cdot (1+z\xi)^3 + (1 - \Omega_m)}}\, dz\xi$$

$$\Phi^2(\chi_{em}) = \begin{cases} \sinh^2 \chi, & k = -1; \\ \chi^2, & k = 0; \\ \sin^2 \chi, & k = +1. \end{cases}$$

photon emitted at time t_{em}

Note: For k = 0

$$\Phi(\chi_{em}) = \chi_{em}$$

Bolometric Flux is the Flux Integrated over Entire Spectrum
Then the Bolometric Magnitude for k=0 is Given by:

$$m_{bol}(z, \Omega_m) := 5\log(1+z) + 5\log\left(\chi_{em}(z, \Omega_m)\right) + 25$$