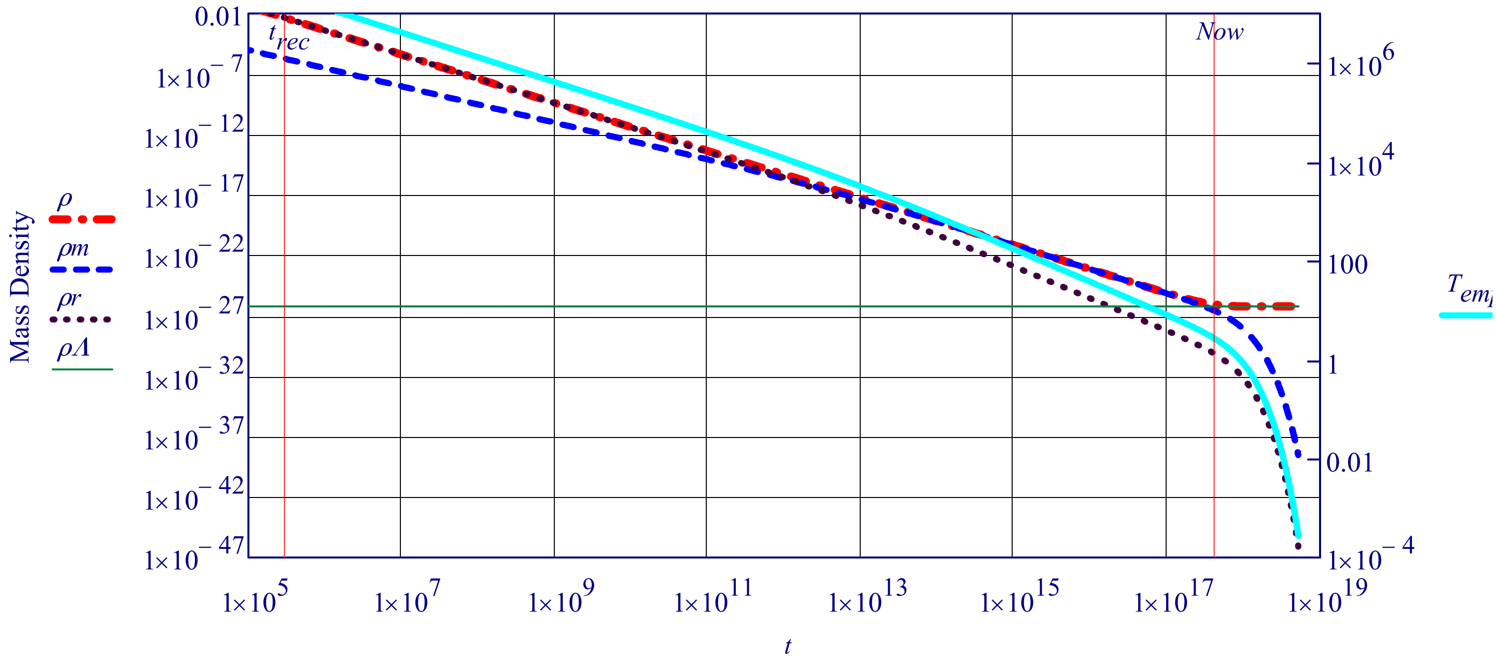
Exploring the Behavior of Some Cosmology Models by Plotting Their Parameters Given by the Definitions in Section VII.

Plots of Cosmic Density Components, Scale Factor, Recession Velocity, Hubble Factor Cosmic Scale Factor, Components of the Energy of the Universe

Recombination Time (s)

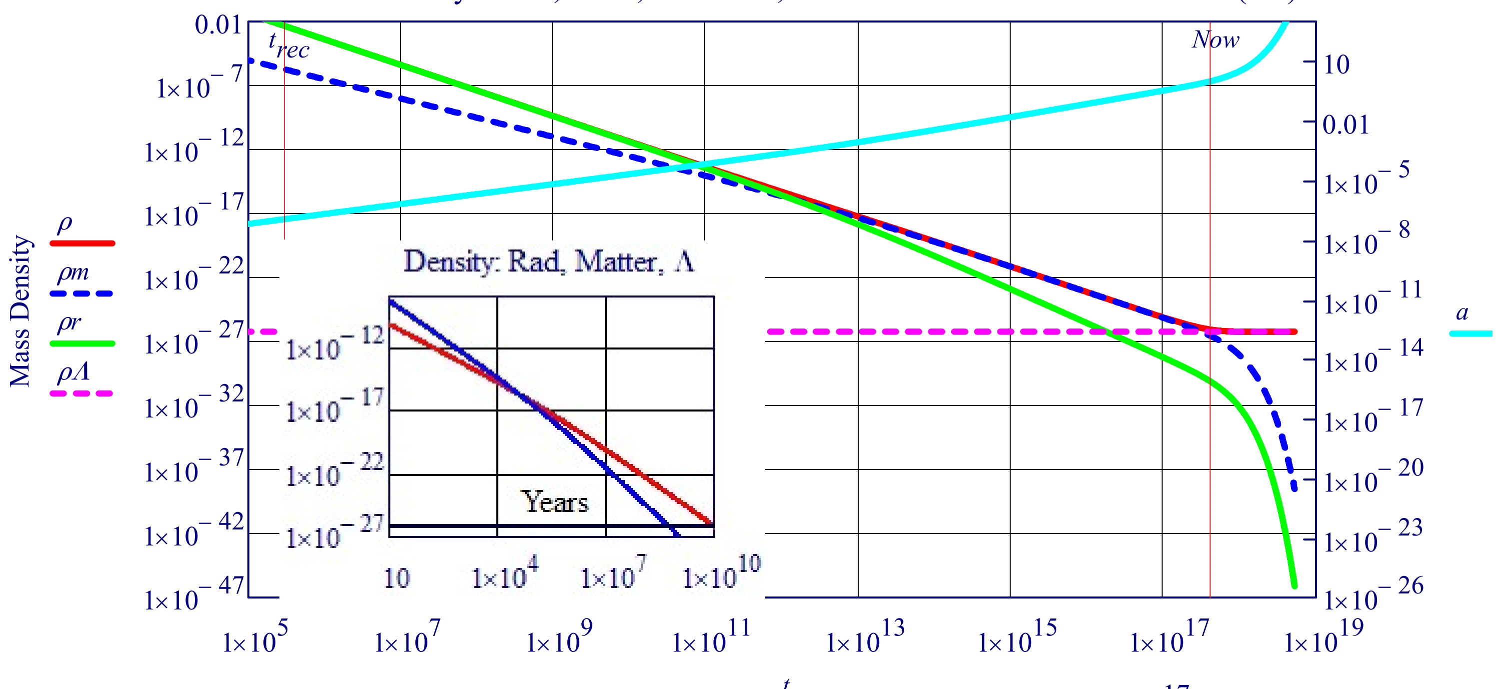
$\rho_0 = 8.644 \frac{kg}{m^3} \cdot 10^{-27}$ $\Omega_{m0} = 0.317$ $\Omega_{r0} = 0$ $\Omega_{\Lambda 0} = 0.683$ $t_{rec} := 3 \cdot 10^5$ $t_{hadron} := 1$ $One := 1$

Cosmic Density: Total, Mass, Radiation, Lambda and Temperature vs Time (sec)



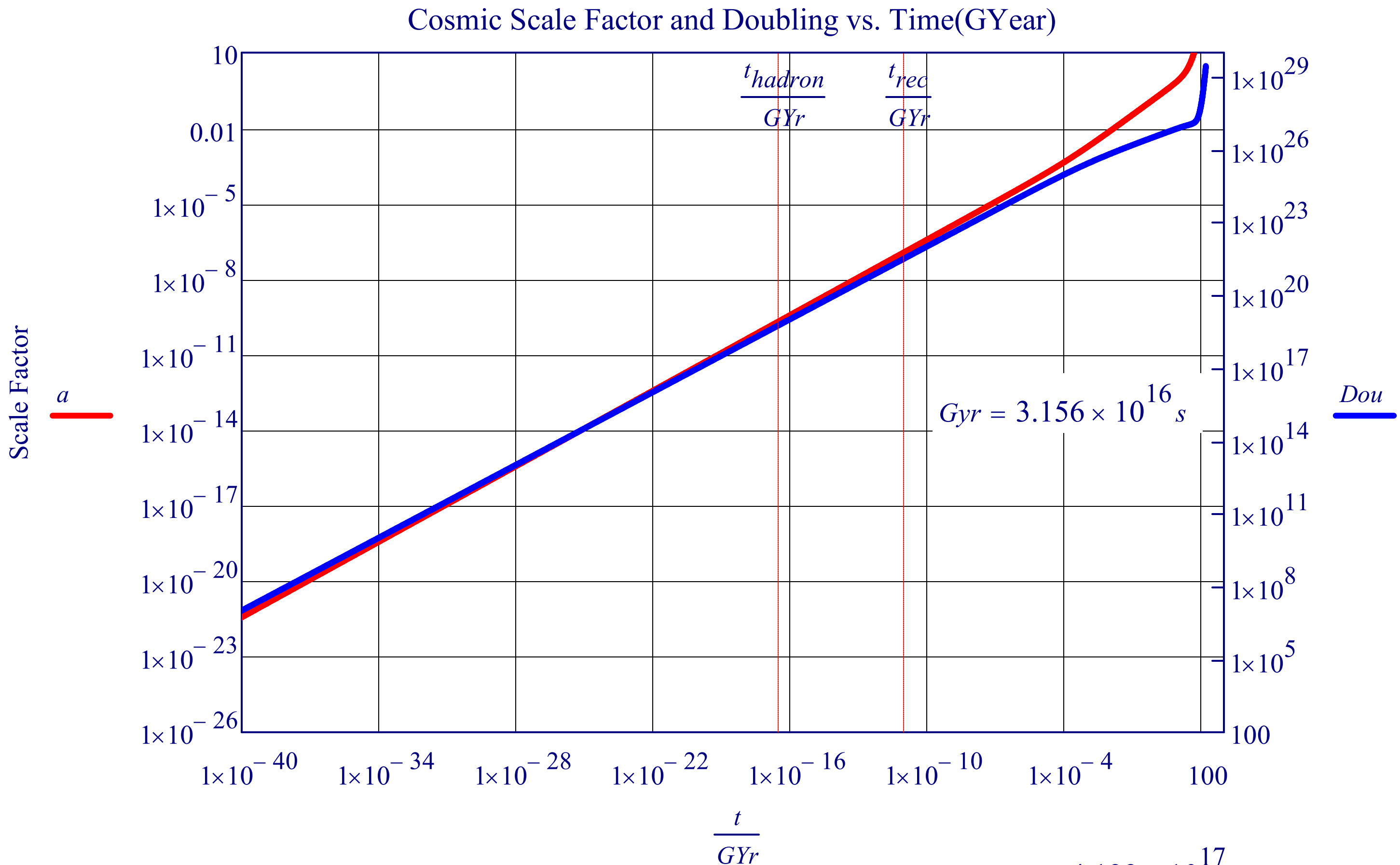
$14Gyr = 4.418 \times 10^{17}\, s$

Cosmic Density: Total, Mass, Radiation, Lambda and Scale Factor vs Time (sec)



$Now = 4.133 \times 10^{17}$

$GYr := \frac{Gyr}{s}$

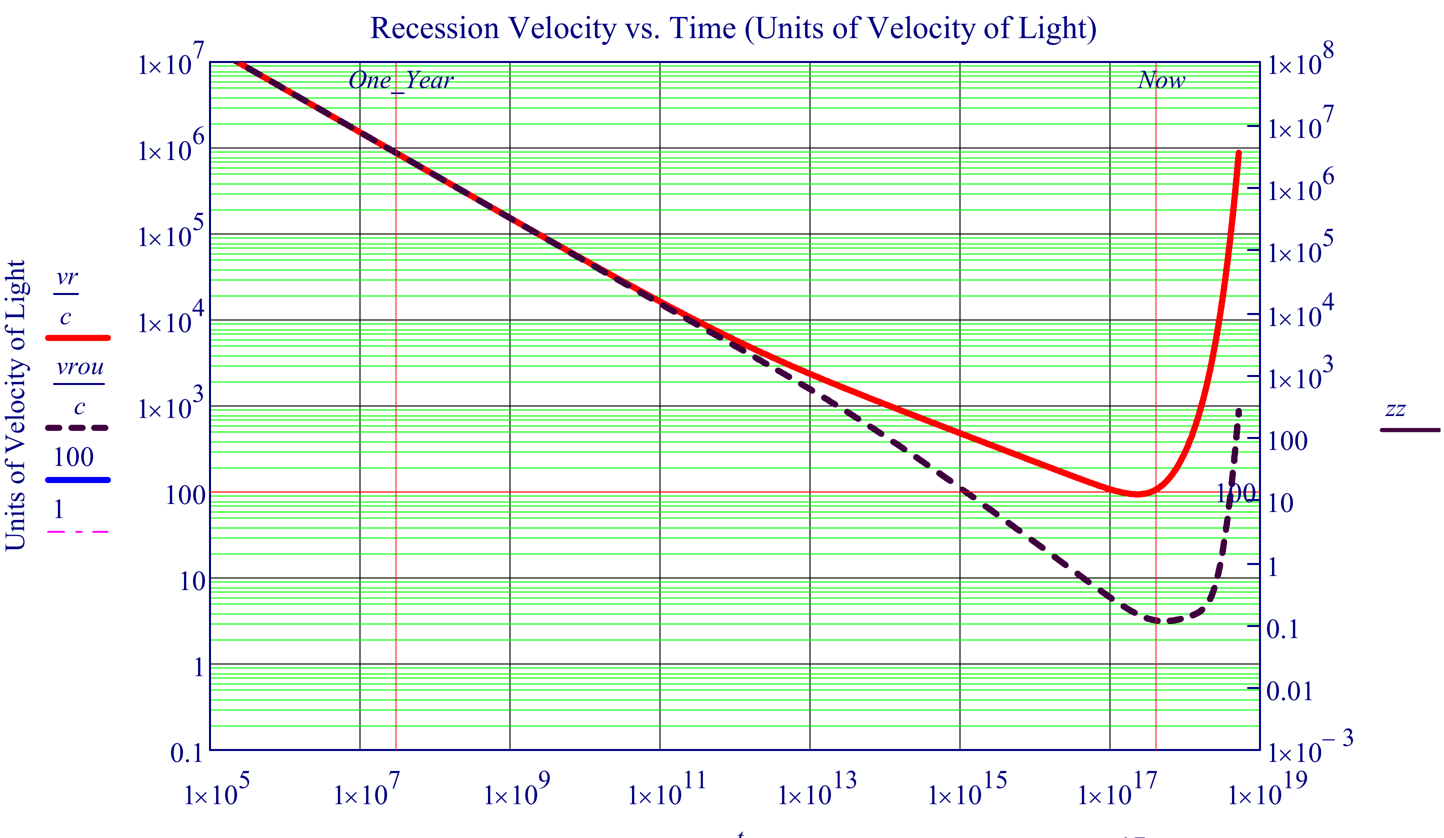


$zz_i := 10^{0.01 \cdot i - 3}$

time (GYear)

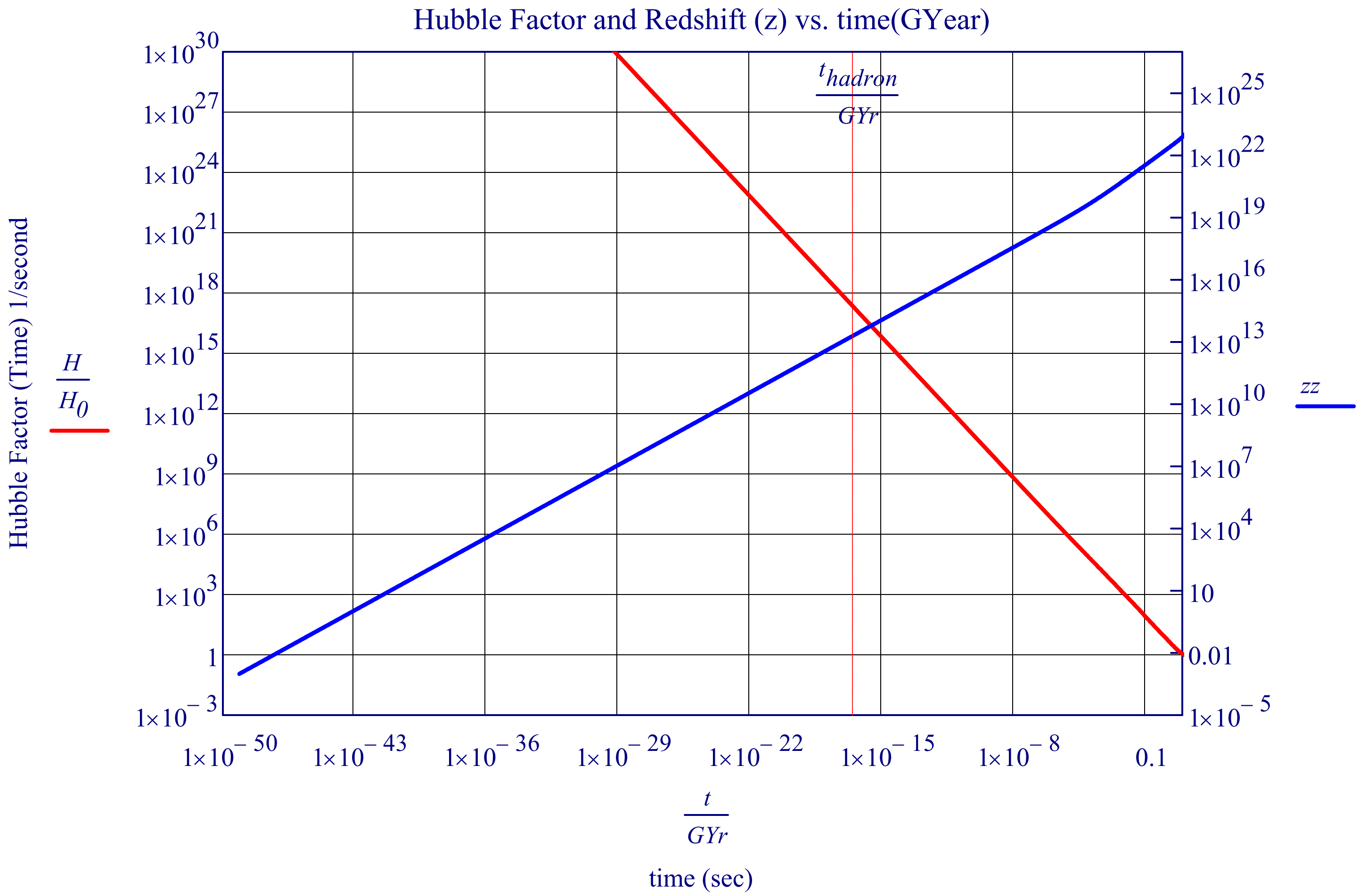
$Now = 4.133 \times 10^{17}$

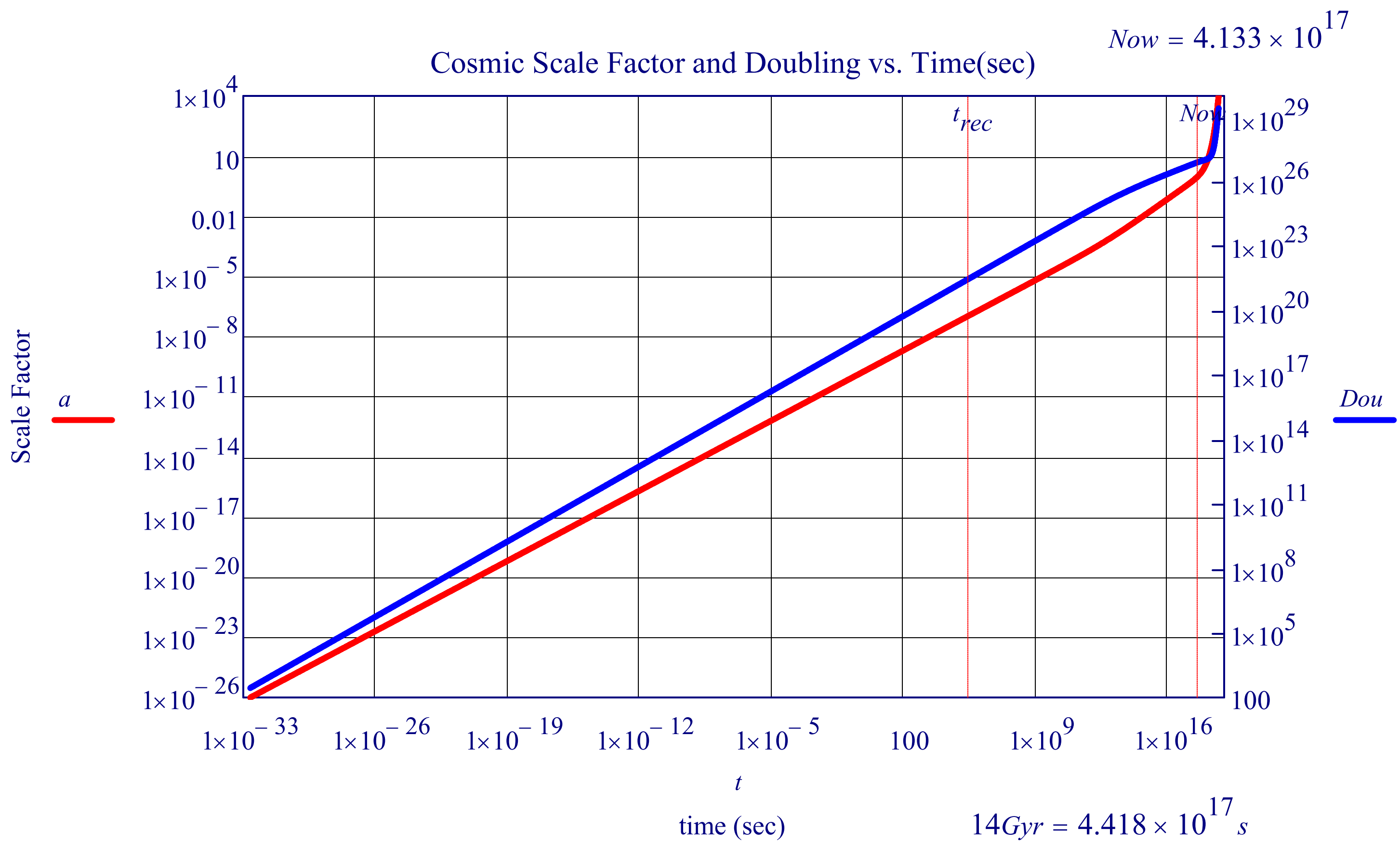
$14Gyr = 4.418 \times 10^{17} s$



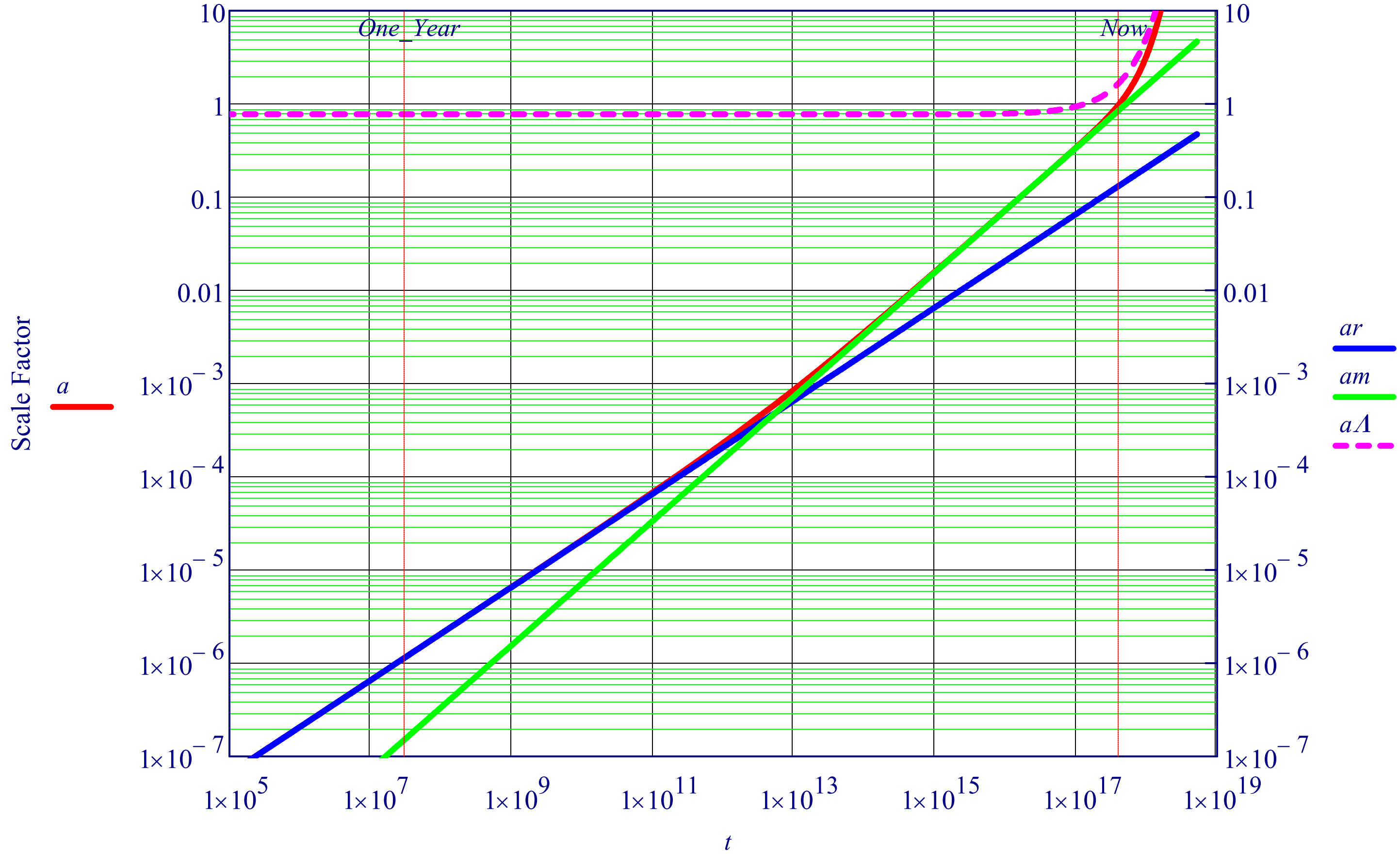
time (sec)

$14Gyr = 4.418 \times 10^{17} s$


Hubble Factor and Redshift (z) vs. time(GYear)
Hubble Factor (Time) 1/second
H / H0
zz
t_hadron / GYr
t / GYr
time (sec)

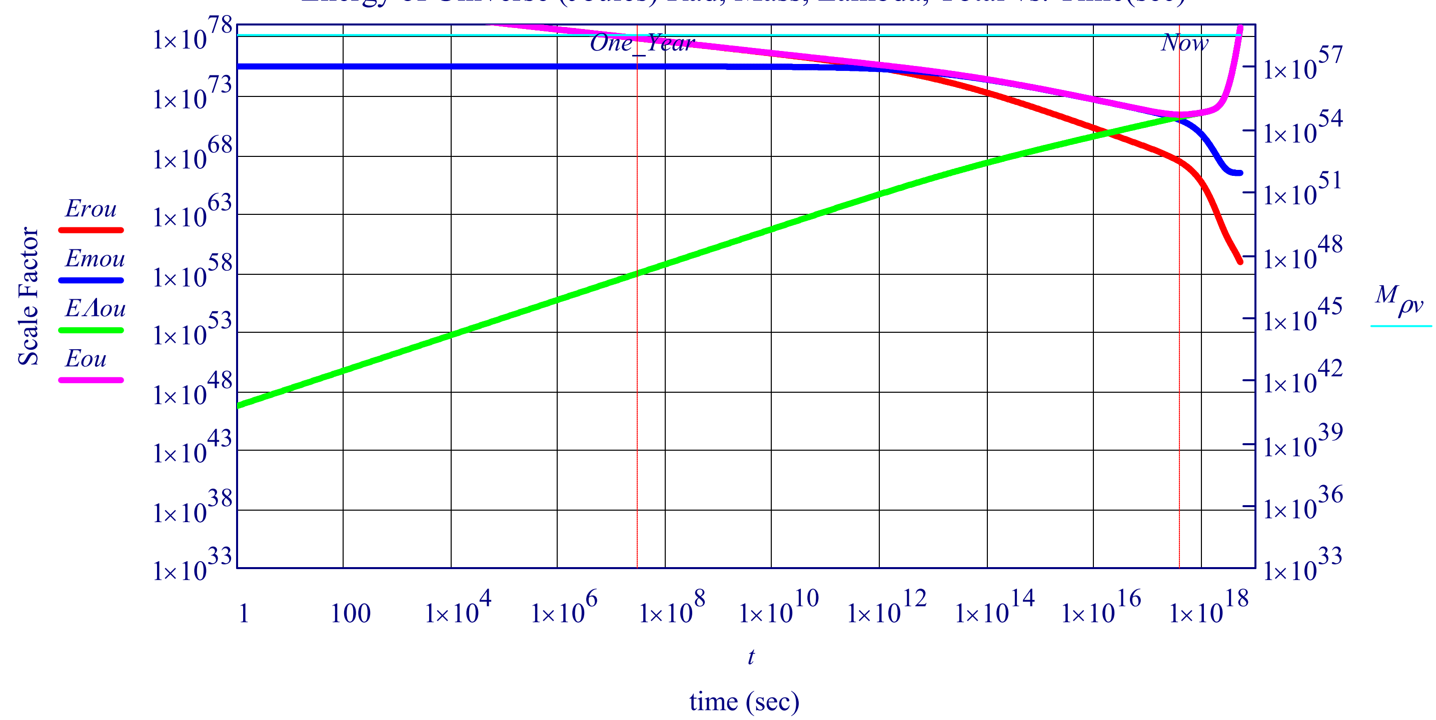

Now = 4.133 × 10^17
Cosmic Scale Factor and Doubling vs. Time(sec)
Scale Factor
a
Dou
t_rec
Now
t
time (sec)
14Gyr = 4.418 × 10^17 s

Cosmic Scale Factor, ar, am, a-Lambda vs. Time(sec)



$14Gyr = 4.418 \times 10^{17}s$

Energy of Universe (Joules) Rad, Mass, Lambda, Total vs. Time(sec)



Plot in GYears

For a radiation-dominated critical density Universe, H0 = 1/2t

$$h_{bar} := 6.62607015 \cdot 10^{-34} m^2 \frac{kg}{2\pi \cdot s}$$

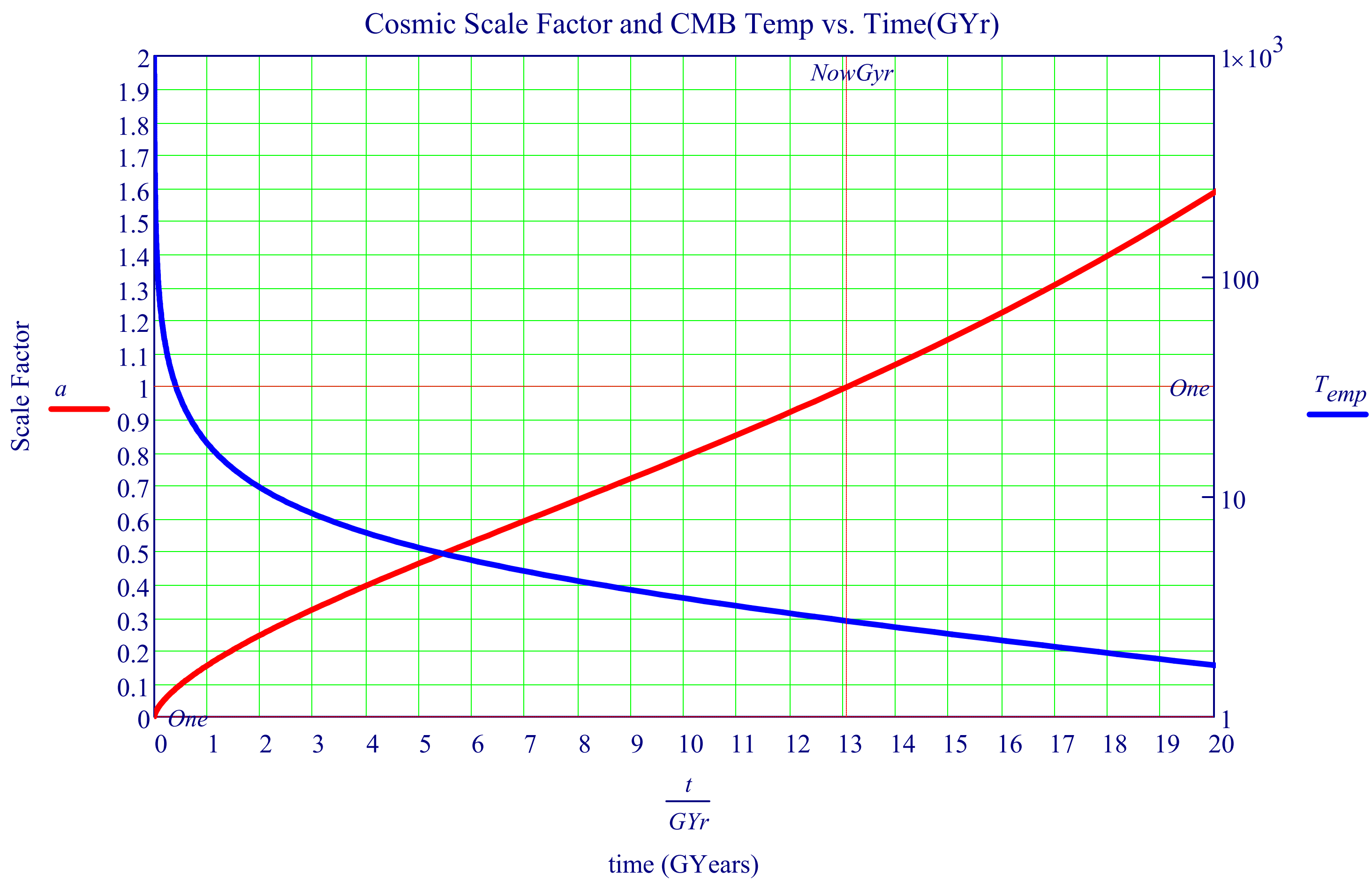
Planck Time, t_{Pl} $\quad t_{Pl} = \frac{h}{2\pi \cdot m_{Pl} \cdot c^2} \quad t_{Pl} := \frac{h_{bar}^{\frac{1}{2}} \cdot G^{\frac{1}{2}}}{c^{\frac{5}{2}}} \quad t_{Pl} = 5.39 s \cdot 10^{-44}$

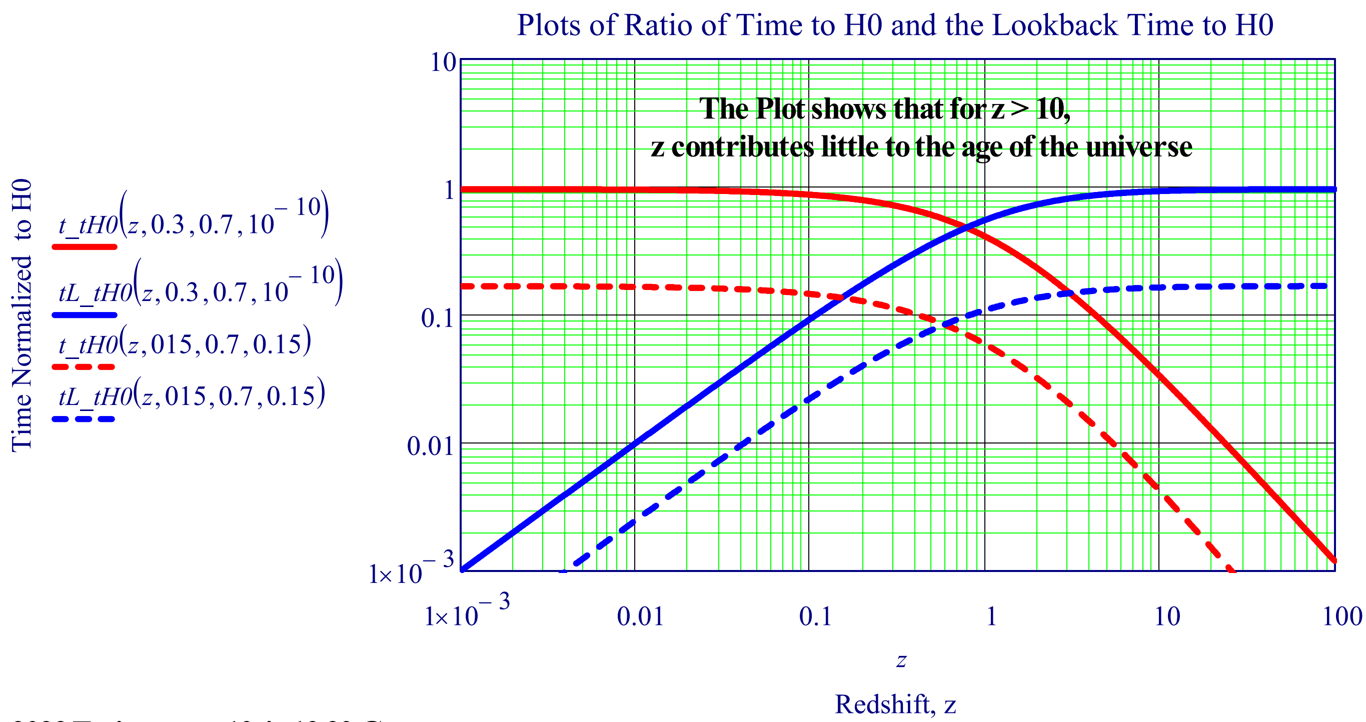
Density at Planck Time, ρ_{crit}

$$\rho_{crit} := \frac{3 \cdot H^2}{8 \cdot \pi G} \qquad H = \frac{1}{2t} \qquad \rho_{time}(t) := \frac{3}{32\pi G \cdot t^2} \qquad \rho_{Pl} := \rho_{time}(t_{Pl}) \qquad \rho_{Pl} = 1.54 \times 10^{95} \frac{kg}{m^3}$$

$$\rho_{1ns} := \frac{3}{32 \cdot \pi \cdot G \cdot \left(10^{-9} s\right)^2} = 4.474 \times 10^{26} \frac{kg}{m^3} \qquad \Delta\rho_{1ns} := \rho_{1ns} + 0.5 \frac{kg}{m^3}$$

$$GYr := 3.156 \cdot 10^{16} \qquad NowGyr := \frac{Now}{GYr}$$





2023 Estimate z=10 is 13.30 Gyr

$t_{BB} := 13.8 Gyr$

$t_{BB} \cdot tL\_tH0\left(10, 0.3, 0.7, 10^{-10}\right) = 12.844 \cdot Gyr$ $\qquad z = \frac{1}{a} - 1$

Dynamics of the expansion

To the observer, the evolution of the scale factor is most directly characterized by the change with redshift of the Hubble parameter and the density parameter; the evolution of ***H(z)*** and ***Ω(z)*** is given immediately by the **Friedmann Equation** in the form $H^2 = 8\pi G\rho/3 - kc^2/R^2$. Inserting the above dependence of $\boldsymbol{\rho}$ on $\boldsymbol{a}$ gives

$$H^2(a) = H_0^2\left[\Omega_v + \Omega_m a^{-3} + \Omega_r a^{-4} - (\Omega - 1)a^{-2}\right].$$

$$H_0 = \left.\frac{\dot{a}}{a}\right|_{t=t_0} \qquad \mathbb{E} \equiv H/H_0 \qquad \mathrm{d}t = \mathrm{d}a/aH$$

This is a crucial equation, which can be used to **obtain the Relation between Redshift and Comoving Distance.**
The radial equation of motion for a photon is $R\,dr = c\,dt = c\,dR/R_{dot} = c\,dR/(RH)$.
With $R = R_0/(1+z)$, this gives

$$\begin{aligned} R_0 dr &= \frac{c}{H(z)}\,dz \\ &= \frac{c}{H_0}\left[(1-\Omega)(1+z)^2 + \Omega_v + \Omega_m(1+z)^3 + \Omega_r(1+z)^4\right]^{-1/2} dz. \end{aligned}$$

This relation is arguably the single most important equation in cosmology,
since it shows how to **relate comoving distance to redshift**, Hubble constant and density parameter.

The comoving distance determines the apparent brightness of distant objects, and the comoving volume element determines the numbers of objects that are observed. These aspects of observational cosmology are discussed in more detail below.

Lastly, using the expression for *H(z)* with $\Omega(a) - 1 = kc^2/(H^2 R^2)$ gives

the redshift dependence of the total density parameter:

$$\Omega(a) - 1 = \frac{\Omega - 1}{1 - \Omega + \Omega_v a^2 + \Omega_m a^{-1} + \Omega_r a^{-2}}.$$

This last equation is very important.

It tells us that, at high redshift, all model universes apart from those with only vacuum energy will tend to look like the $\Omega = 1$ model.

This is not surprising given the form of the Friedmann equation: provided $\rho R^2 \to \infty$ as $R \to 0$, the $-kc^2$ curvature term will become negligible at early times.
If $\Omega \neq 1$, then in the distant past $\Omega(z)$ must have differed from unity by a tiny amount: the density and rate of expansion needed to have been finely balanced for the universe to expand to the present.

This tuning of the initial conditions is called the flatness problem and is one of the motivations for the applications of quantum theory to the early universe.

Evolution of the Hubble Factor: Mass Conservation of non-relativistic matter implies $\rho_m \propto a^{-3} = (1+z)^3$.
In the ΛCDM model, dark energy is assumed to behave like a cosmological constant: $\rho_\Lambda \propto a^0 = (1+z)^0$.
The density of radiation (and massless neutrinos) scales as $\rho_r \propto a^{-4} = (1+z)^4$ because the number density of photons is $\propto a^{-3} = (1+z)^3$ and the mass $E/c^2 = h\nu/c^2$ of each photon scales as $E \propto \lambda^{-1} \propto (1+z)^1 \propto a^{-1}$.

Calculated Values

$\Omega_{r0} = 0$
$\Omega_{m0} = 0.317$
$\Omega_{\Lambda 0} = 0.683$

Dynamical Equation Specifying the Evolution of the Hubble Factor of Our Universe

$$\frac{H}{H_0} = H\_H_0(z) := \sqrt{\Omega_{m0}\cdot(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{r0}\cdot(1+z)^4}$$

***R(t)* is the Scale Factor**

$R(t_0) = 1$

$$H(t) = \frac{\frac{d}{dt}R(t)}{R(t)}$$