VIII. Multiple-Component Universes: Parameter (t_0H_0) Contour Vs. Densitites

ASTROPHYSICS AND COSMOLOGY

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Define $y = \frac{a}{a_0}$ $\qquad \tau = H_0 \cdot (t - t_0)$

Then Friedmann's Equation can be written:

$$\frac{d}{d\tau} y = \sqrt{1 + \left(\frac{1}{y} - 1\right) \cdot \Omega_M + \left(y^2 - 1\right) \cdot \Omega_\Lambda}$$

Equation 56

With Initial Conditions

$$y(0) = 1 \qquad \frac{d}{d\tau} y(0) = 1$$

Therefore, the present age t_0 is a function of the other parameters,

$t_0 = f(H_0, \Omega_M, \Omega_\Lambda)$, determined from

$$t0H0\left(\Omega_M, \Omega_\Lambda\right) := \int_0^1 \frac{1}{\sqrt{1 + \left(\frac{1}{y} - 1\right) \cdot \Omega_M + \left(y^2 - 1\right) \cdot \Omega_\Lambda}} \, dy$$

$t0H0(0.3, 0.7) = 0.964$

$\dot{a}^2 = H_0^2[\Omega_m a^{-1} + (1 - \Omega_m)a^2]$ and the time relationship $H_0 t(a) = \int_0^a \frac{x \, dx}{\sqrt{\Omega_m x + (1 - \Omega_m)x^4}}$

Calculate a Matrix Time$_0$H$_0$ (t0H0) of Values:

of t0H0 for Ω_M and Ω_Λ Ranging from 0 to 1.5

```
Time0H0 := TML ← (0  0  0)
           m ← 0
           l ← 0
           for  mm ∈ 0, 1 .. 170
               m ← m + 0.01
               l ← 0
               for  ll ∈ 0, 1 .. 100
                   th ← t0H0(m, l)
                   tml ← (m  l  th)
                   l ← l + 0.01
                   TML ← stack(TML, tml)
           TML
```

$min\left(Time0H0^{\langle 0 \rangle}\right) = 0$

$min\left(Time0H0^{\langle 1 \rangle}\right) = 0$

$min\left(Time0H0^{\langle 2 \rangle}\right) = 0$

$max\left(Time0H0^{\langle 0 \rangle}\right) = 1.71$

$max\left(Time0H0^{\langle 1 \rangle}\right) = 1$

$max\left(Time0H0^{\langle 2 \rangle}\right) = 2.062$

$rows(Time0H0) = 17272$

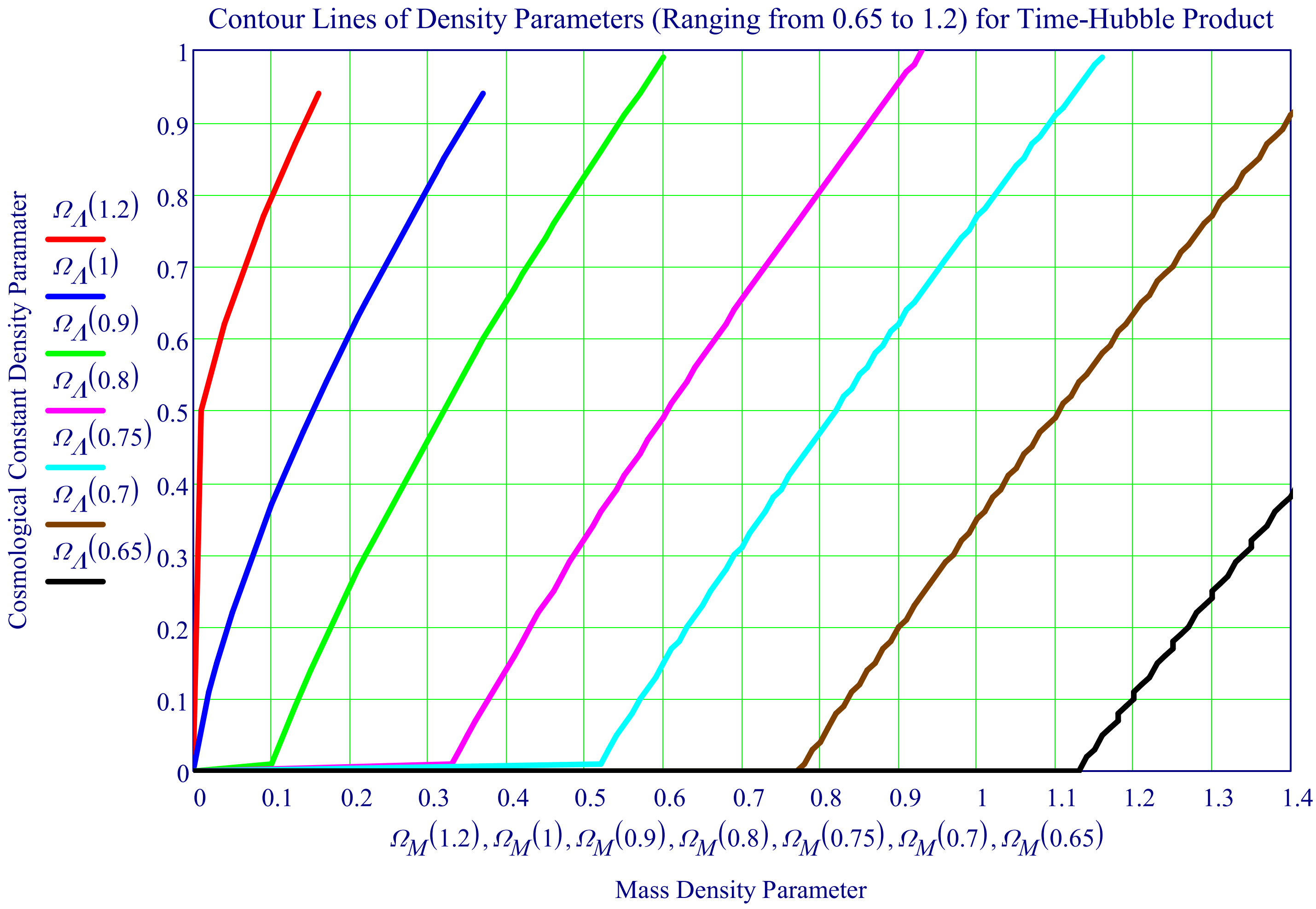
Assemble Contour Line Points of Curves with Given t_0H_0 Values

Find Those Contour Values of **Density Parameters**, Ω_M and Ω_Λ,
of Matrix $Time_0H0$ that Give a t_0H_0 values (T) ranging from 0.65, 0.7 ... up to 1.2

```
TH(T) :=  | R ← 0
          | TH ← (0  0  0)
          | for  r ∈ 0, 1 .. 17000
          |   if  (Time0H0_{r,2} < T + 0.001) ∧ Time0H0_{r,2} > T − 0.00
          |       | out ← (Time0H0_{r,0}  Time0H0_{r,1}  Time0H0_{r,2})
          |       | TH ← stack(TH, out)
          | TH
```

$\Omega_\Lambda(T) := TH(T)^{\langle 1 \rangle}$ $\qquad$ $\Omega_M(T) := TH(T)^{\langle 0 \rangle}$ $\qquad$ $t0H0(1,0) = 0.667$

$t0H0(0.01,1) = 2.062$

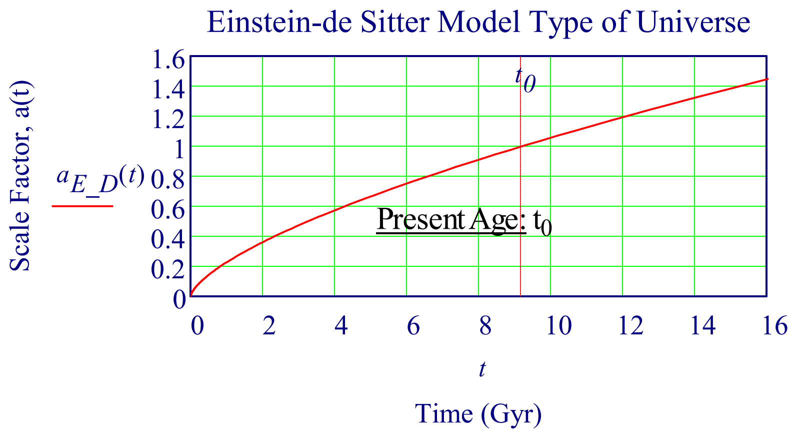


Einstein-de Sitter (EdS) Model Universe: Flat and Matter-Only FLRW Universe

The Einstein–de Sitter universe is a model of the universe proposed by Albert Einstein and Willem de Sitter in 1932. On first learning of Edwin Hubble's discovery of a linear relation between the redshift of the galaxies and their distance, **Einstein set the cosmological constant to zero** in the Friedmann equations, resulting in a model of the expanding universe known as the Friedmann–Einstein universe. In 1932, Einstein and De Sitter proposed an even simpler cosmic model by assuming a **vanishing spatial curvature as well as a vanishing cosmological constant**. In modern parlance, the Einstein–de Sitter universe can be described as a **Cosmological Model for a Flat Matter-Only** Friedmann–Lemaître–Robertson–Walker metric (FLRW) universe.

In the model, Einstein and de Sitter derived a simple relation between the average density of matter in the universe and its expansion according to $H_0^2 = \kappa\rho/3$, where H_0 is the Hubble constant, ρ is the average density of matter and κ is the Einstein gravitational constant. The cosmic time t as a function of scale factor, a, is given by

$$a_{eds}(t) := c\cdot e^{\sqrt{\frac{8\pi\cdot G\cdot \rho_0}{3}\cdot t}} \qquad t_0 := \frac{2}{3\cdot H_0} = 9.2\cdot Gyr \qquad t_0 := \frac{t_0}{Gyr} \qquad a_{E\_D}(t) := \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

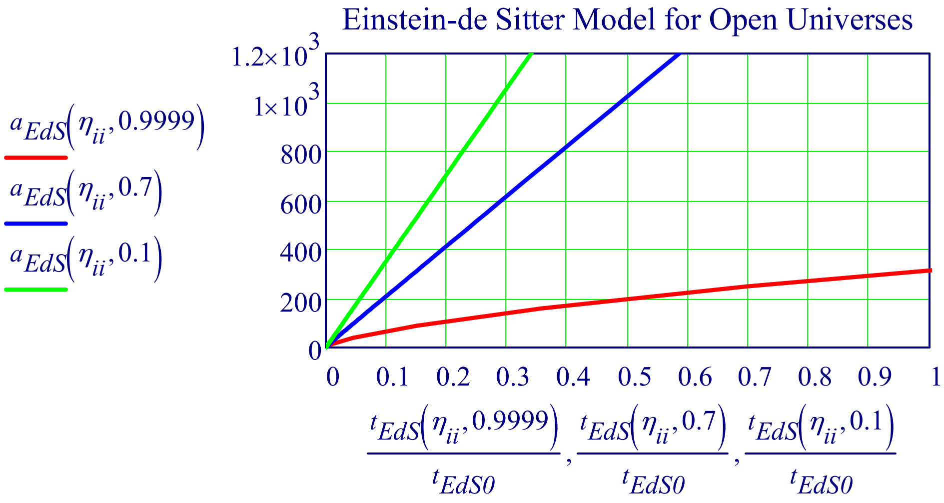


$t_0 = 9.2$

EdS: The cosmic time t as a function of the scale factor, a, is given by the Expression:

$$a_{EdS}(\eta, \Omega_0) := \frac{1}{2}\cdot\frac{\Omega_0}{1-\Omega_0}\cdot(\cosh(\eta) - 1) \qquad t_{EdS}(\eta, \Omega_0) := \frac{1}{2H_0\cdot\frac{Gyr\cdot km}{Mpc}}\cdot\frac{\Omega_0}{(1-\Omega_0)^{\frac{3}{2}}}\cdot(\sinh(\eta) - \eta)$$

$ii := 0..200$ $\qquad \eta_{ii} := \frac{2\cdot\pi\cdot ii}{100}$ $\qquad$ Time Normalized to $\Omega_0 = 0.9$ $\qquad t_{EdS0} := t_{EdS}(2\pi, 0.9)$



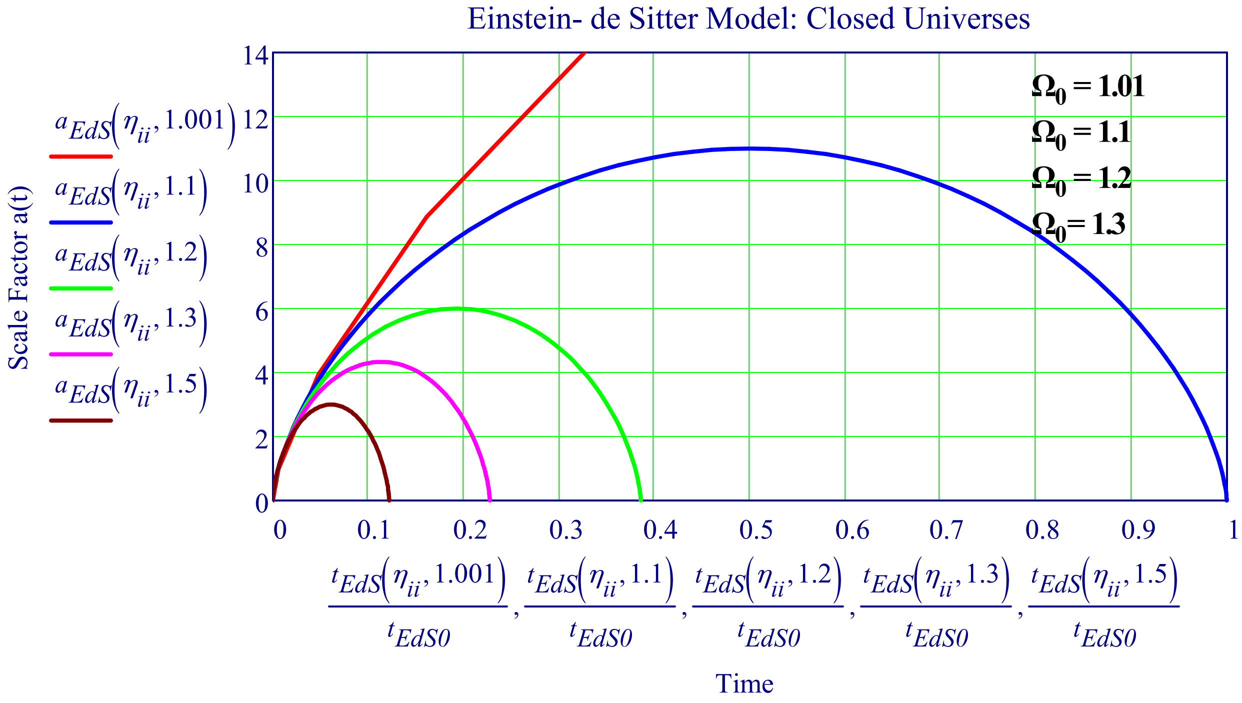
Plots of *a(t)* versus t for the closed universes with $\Omega_0 = 1.1, 1.2, 1.5,$

$$a_{EdS}(\eta,\Omega_0) := \frac{1}{2}\cdot\frac{\Omega_0}{\Omega_0-1}\cdot(1-\cos(\eta))$$

$$t_{EdS}(\eta,\Omega_0) := \frac{1}{2H_0\cdot\frac{Gyr\cdot km}{Mpc}}\cdot\frac{\Omega_0}{(\Omega_0-1)^{\frac{3}{2}}}\cdot(\eta-\sin(\eta))$$

$ii := 0..100$ $\qquad \eta_{ii} := \frac{2\cdot\pi\cdot ii}{100}$ $\qquad t_{EdS0} := t_{EdS}(2\pi, 1.1)$

See Section XXXII on the Fine Tuning Flatness Problem



Temperature Jumps at Phase Transitions. Temperature at Recombination, E_{th}.

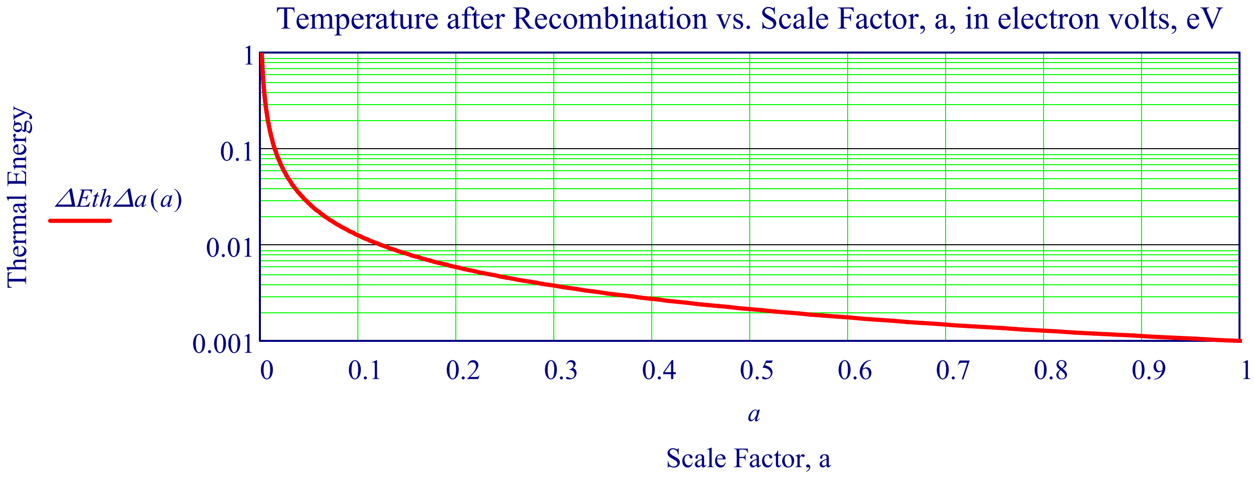
A New Version of the Lambda-CDM Cosmological Model, with Extensions and New Calculations,
Journal of Modern Physics, 2024, 15, 193-238, Jan Helm

$\eta := 1.1$

$E_{th0} := -0.001$

Rate of Change of Eth with scale factor a, ΔEthΔa

$\Delta Eth\Delta a = \frac{d}{da}Eth$ $\qquad \Delta Eth\Delta a(a) := \frac{-E_{th0}}{a^{\eta}}$ $\qquad T_{eV} := \frac{1eV}{k_B}$ $\qquad T_{eV} = 1.2\times 10^{4}K$



Measuring Cosmological Parameters

Cosmologists would like to know the scale factor *a(t)* for the universe. For a model universe whose contents are known with precision, the scale factor can be computed from the Friedmann equation. Finding *a(t)* for the real universe, however, is much more difficult. The scale factor is not directly observable; it can only be deduced indirectly from the imperfect and incomplete observations that we make of the universe around us. If we knew the **Energy Density** *ε* for each component of the universe, we could use the Friedmann equation to find the **scale factor *a(t)***. The argument works in the **other direction, as well**; if we could determine *a(t)* from observations, we could use that knowledge to find ε for each component. Let's see, then, what constraints we can put on the scale factor by making observations of distant astronomical objects.

Since **determining the exact functional form of a(t) is difficult**, it is useful, instead, to do a **Taylor series expansion** for a(t) around the present moment. Keeping the first three terms of the Taylor expansion, the scale factor in the recent past and the near future can be approximated as

$$a(t) = a(t_0) + \left.\frac{da}{dt}\right|_{t=t_0}(t-t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_0}(t-t_0)^2 + \cdots$$

Using the normalization $a(t_0) = 1$, the expansion can be written: $$a(t) \approx 1 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2$$

the parameter q_0 is a dimensionless number called the deceleration parameter, defined as

$$q_0 \equiv -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t=t_0} = -\left(\frac{\ddot{a}}{aH^2}\right)_{t=t_0}$$

Although H_0 and q_0 are themselves free of the theoretical assumptions underlying the Friedmann and acceleration equations, we can use the acceleration equation to predict what q0 will be in a given model universe. If our model universe contains N components, each with a different value of the equation-of-state parameter w_i, the acceleration equation can be written

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\sum_{i=1}^{N}\varepsilon_i(1+3w_i) \qquad -\frac{\ddot{a}}{aH^2} = \frac{1}{2}\left[\frac{8\pi G}{3c^2H^2}\right]\sum_{i=1}^{N}\varepsilon_i(1+3w_i)$$

The relation between the deceleration parameter q_0 and the density parameters of thedifferent components of the universe **For the current BB Model:**

$$q_0 = \frac{1}{2}\sum_{i=1}^{N}\Omega_{i,0}(1+3w_i) \qquad q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0} \qquad q_0 := 0.53$$

1. In principle, determining H_0 should be easy. For small redshifts, the relation between a galaxy's distance d and its redshift z is linear Equation: $cz = H_0 d$ where $z = 1/a(t_e) - 1$

Thus, if you measure the distance d and redshift z for a large sample of galaxies, and fit a straight line to a plot of cz versus d, the slope of the plot gives you the value of H_0. In practice, the distance to a galaxy is not only difficult to measure, but also somewhat difficult to define. The proper distance *dp(t)* between two points was defined as the length of the spatial geodesic between the points when the scale factor is fixed at the value a(t). The proper distance is perhaps the most straightforward definition of the spatial distance between two points in an expanding universe. We can get an approximate form by taking the first two terms of the Taylor expansion.

$$d_p(t_0) = c\int_{t_e}^{t_0}\frac{dt}{a(t)} \qquad d_p(t_0) \approx c(t_0-t_e) + \frac{cH_0}{2}(t_0-t_e)^2$$

where $c\,(t_0 - t_e)$ is the proper distance in a static universe.

substituting the *dp(t₀)* equation into the Taylor Expansion gives:

$$d_p(t_0) \approx \frac{c}{H_0}\left[z - \left(1+\frac{q_0}{2}\right)z^2\right] + \frac{cH_0}{2}\frac{z^2}{H_0^2} = \frac{c}{H_0}z\left[1 - \frac{1+q_0}{2}z\right]$$

Light-cone structure of the FLRW space

$$ds^2 = -c^2dt^2 + a^2(t)\left(\frac{dr^2}{1-Kr^2} + r^2d\Omega^2\right)$$

Let us consider the K = 0 case, for simplicity. Moreover, consider also ***$d\Omega = 0$***. In this case, the radial coordinate is also the distance. Then, putting ***$ds^2 = 0$*** in the FLRW metric gives the following light-cone structures.

Cosmic time-comoving distance

From the above FLRW metric, the condition ***$ds^2 = 0$*** gives us:

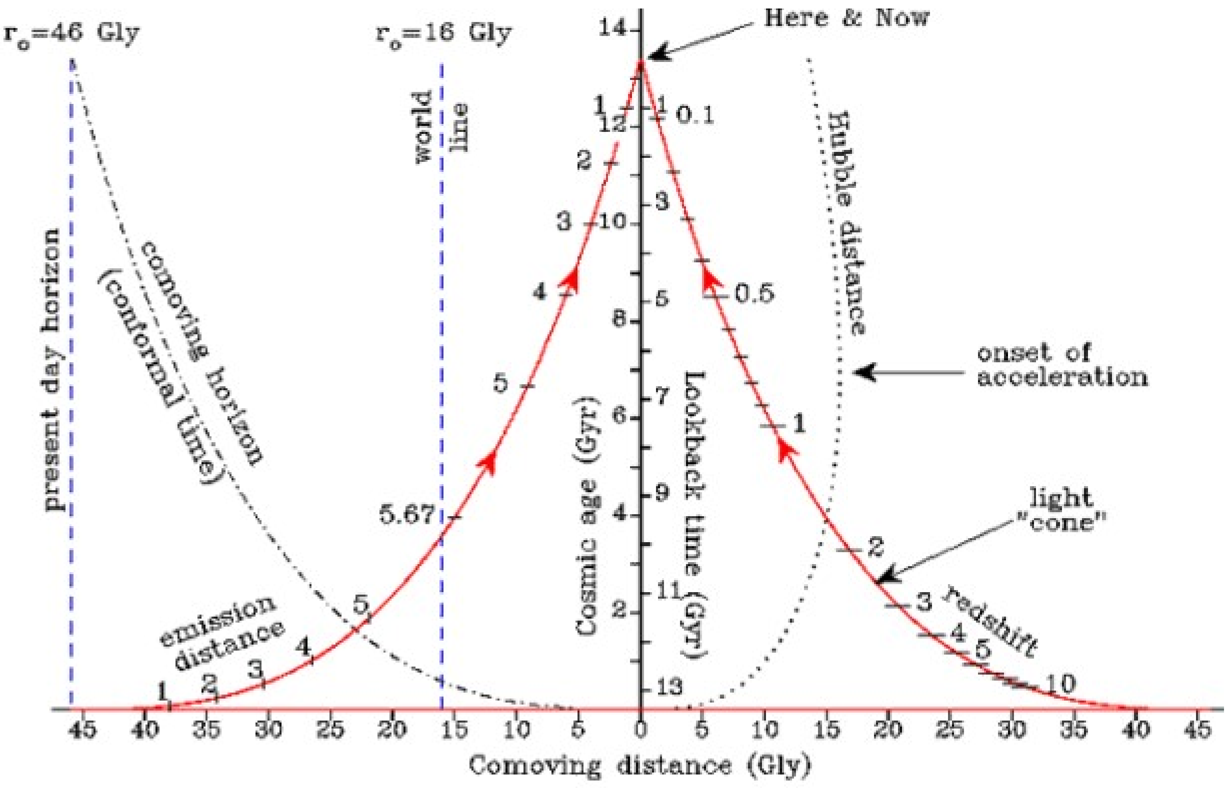
$$\frac{cdt}{dr} = \pm a(t)$$

We put our observer at r = 0 and $t = t_0$. The plus sign in the above equation then describes an outgoing photon, i.e. the future light-cone, whereas the negative sign describes an incoming photon, i.e. the past-light cone, which is much more interesting to us. So, let us keep the negative sign and discuss the shape of the light-cone. Assume that a(0) = 0. Therefore, the slope of the past light-cone starts as $-a(t_0)$, which we can normalise as −1, i.e. locally the past light-cone is identical to the one in Minkowski space. However, a goes to zero, so the light-cone becomes flat, encompassing more radii than it would for Minkowski space. See Figure below . We can show this analytically by taking the second derivative of the above with the minus sign.

$$\frac{c^2d^2t}{dr^2} = -\dot{a}\frac{cdt}{dr} = a\dot{a}$$

Space Time Diagram Comoving Distance and Normal Time

Cosmic Age/Lookback Time - Comoving distance



Space-time diagram and light-cone structure for the FLRW metric