IXB.The Scale of the Universe

The Hubble Length, D_H = equals c/H_0, and the Hubble time, t_H equals $1/H_0$, gives the approximate spatial and temporal scales of the universe.

H_0 is a scale parameter and is independent of the "shape parameters" (expressed as density parameters) Ω_m, Ω_Λ, Ω_k w, etc., which govern the global geometry and dynamics of the universe.

Distances to galaxies, quasars, etc., scale linearly with H_0, $D \approx c\,z\,/H_0$. They are necessary in order to convert observable quantities for example, fluxes, angular sizes into physical ones (luminosities, linear sizes, energies, masses, etc.)

Distance Ladder: Methods

Methods yielding absolute distances:

Parallax (trigonometric. secular. and statistical)
The moving cluster method - has some assumptions
Baade-Wesselink method for pulsating stars
Expanding photosphere method for Type ll SNe Mfidel
Sunyaev-Zeldovich effect <== Model dependent!d
Gravitational lens time delays <==Model dependent!

Telescope Resolution:
Hubble 0.05 arcseconds
Very-long-baseline interferometry (VLBI) 25 μarcsecs

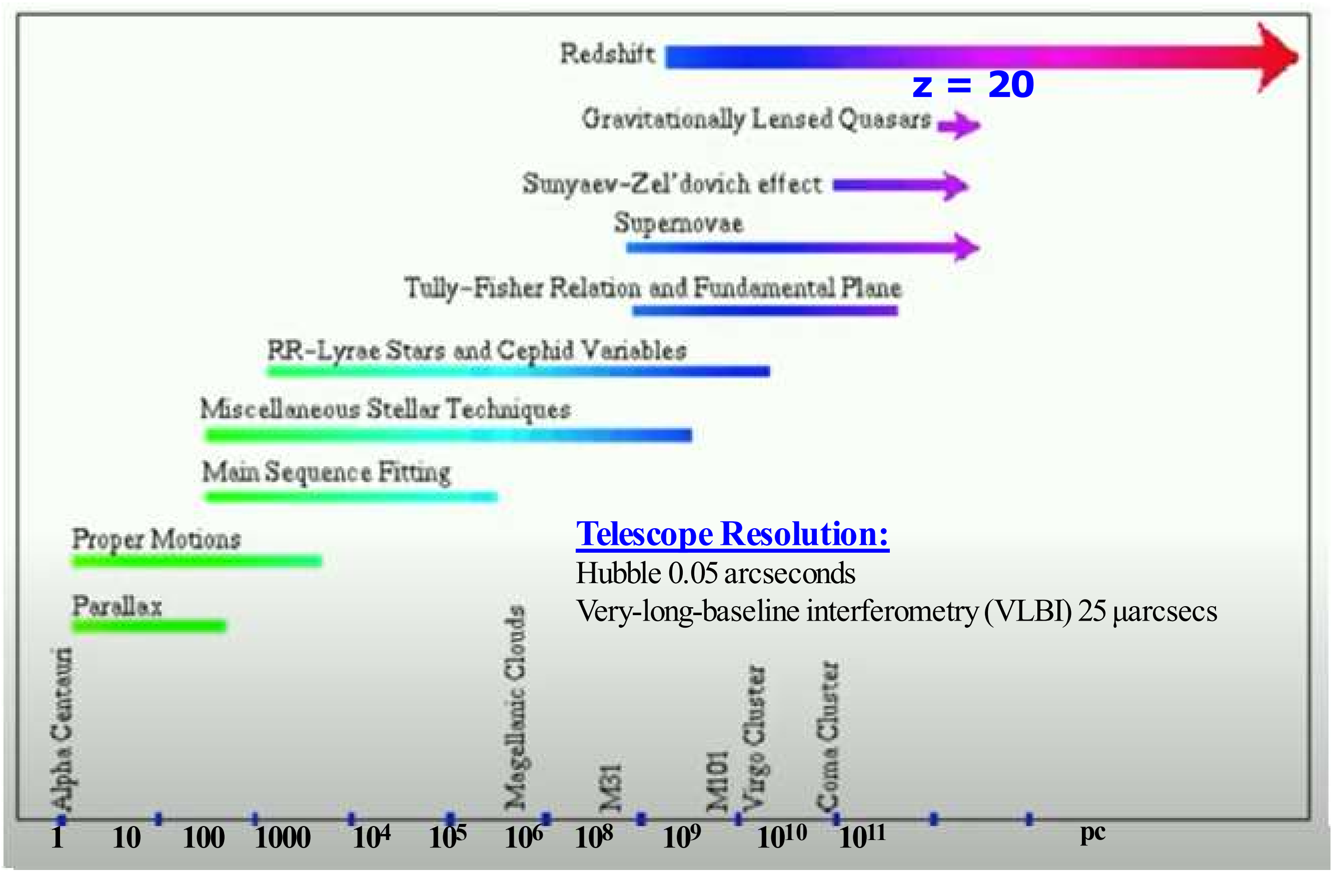
Secondary distance indicators:

"**standard candles**, **requiring a calibration** from an absolute method applied to local objects -

The Distance Ladder:

Pulsating variables: Cepheids. RR Lyrae.
Main sequence titling to star clusters, Brightest red giants
Planetary nebula luminosity function
Globular cluster luminosity function
Surface brightness fluctuations
Tully-Fisher, D_a - σ, FP scaling relations for galaxies
Type 1a Supernovae

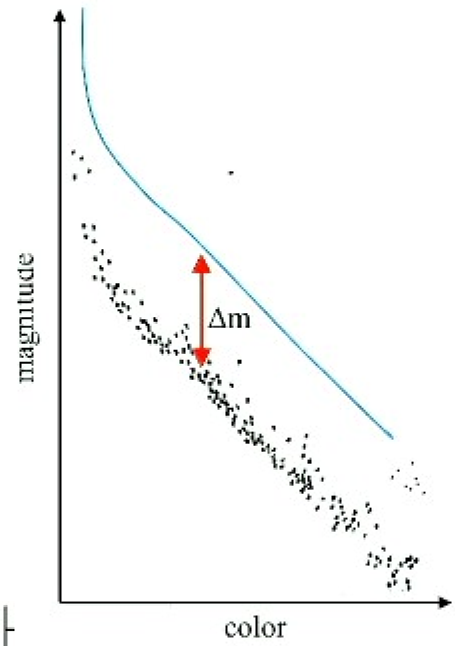
Diameter of our Local Group of galaxies is ≈ 3 megaparsecs, and it contains at least 80 galaxies, most of which are dwarf galaxies



Main Sequence Fitting for Star Clusters

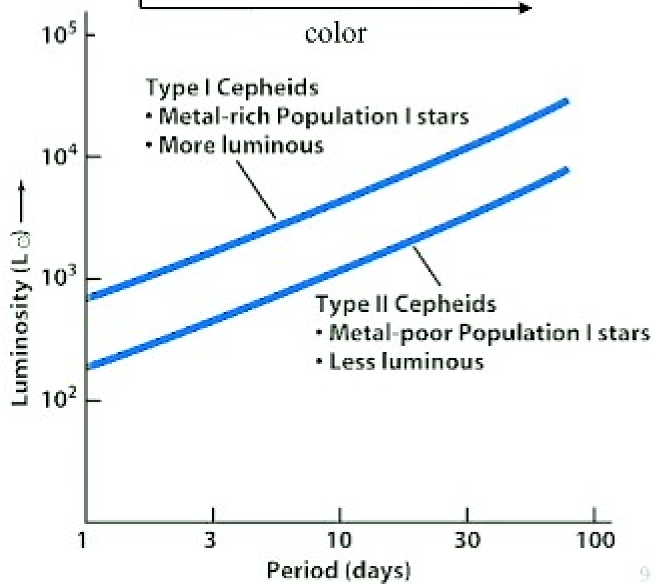
Luminosity (distance dependent) vs. temperature or color (distance independent)

- Can measure distance to star clusters (open or globular) by fitting their main sequence with clusters with known distances from Gaia.
- The apparent magnitude difference gives the ratio of distances, as long as we know the reddening(extinction)!
- For globular clusters we use parallaxes to nearby subdwarfs (metal-poor main sequence stars)



Pulsating Variables

- Stars in the instability strip in the HR diagram.
- All obey empirical period - luminosity distance independent vs.dependent) relations which can be calibrated to yield distances.
- Different types (in different branches of the HRD, different stellar populations) have different relation.
- Cephelds are high-mass, luminous, upper MS, Pop. I stars.
- RR Lyrae are low-mass, rnetal-poor (Pop. II), HB stars, often found in globulars.
- Long-period variables (e.g., Miras) pulse in a fashion that is less "well understood."



Cepheids

- Luminous (M ≈ -4 to -7 mag), pulsating variables high mass stars on the instability strip in the H-R diagram. Henrietta Leavitt (1912) found in a period-luminosity relation for Cepheids in the SMC: brighter ones have longer periods

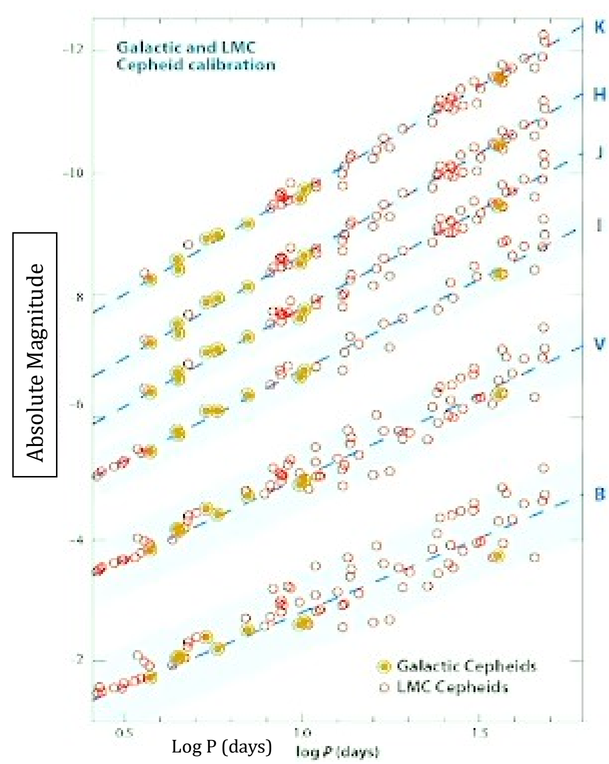
- **Advantages**:

Bright and easily seen in galaxis(out to ≈25 Mpc with the HST, stellar plusation is well understood.

- **Disadvantages**:

Relatively rare, period may depend en rnetallicity or color, need runlet epoch observations found near star forming regions, so extinction corrections are necessary.

- Redder bands have smaller scatter, but also shallower slope.
- Calibrated using parallaxes on the II-R diagram



The Baade-Wesselink Method

$$L = \sigma R^2 T^4$$

Luminosity from the Stephan-Boltzmann formula
Consider a pulsating star at a minimum, with a measured temperature T_1 and observed flux f_1 with radius R_1, then:

$$f_1 = \frac{4\pi R_1^2 \sigma T_1^4}{4\pi D^2}$$

At a maximum, with a measured temperature T_2 and observed flux f2 with radius R2

$$f_2 = \frac{4\pi R_2^2 \sigma T_2^4}{4\pi D^2}$$

Note: T_1, T_2, f_1, f_2, are directly observable! Just need the radius. So, from spectroscopic observations we can get the photospheric velocity v(t), from this we can determine the change in the radius are

$$R_2 = R_1 + \Delta R = R_1 + \int_{t_1}^{t_2} v(t)\, dt$$

3 equations, 3 unknowns, solve for R1, R2, and D
Difficulties: the effects of the stellar atmospheres (not a perfect black body), and deriving the true radial velocity from what we observed.

Galaxy Scaling Relations

Once a set of distances to galaxies of some type is obtained, one finds correlations between distance-dependent quantities that is, luminosity, radius and distance-independent ones, for example, rocational speeds for discs, or velocity dispersions from ellipticals and bulges, surface brightness, etc.
These are called **distance indicator relations**. Examples:

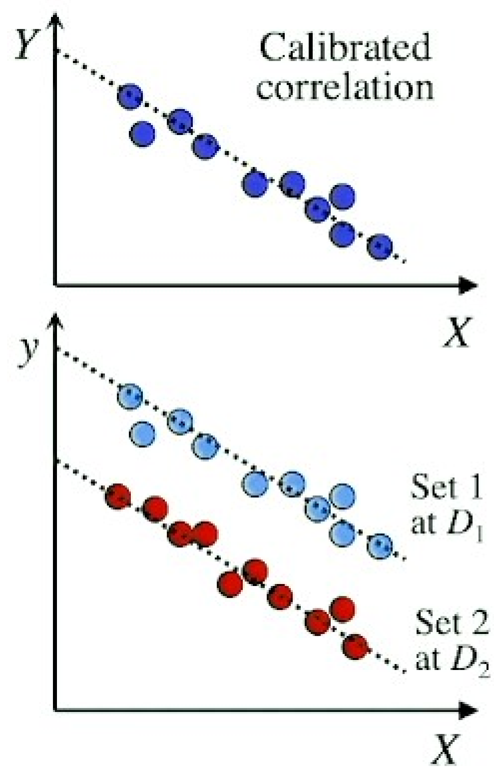
- **Tully-Fisher relation** for spirals (luminosity versus rotation speed).
- **Fundamental Plane** relationships from ellipticals radius versus a combination of velocity dispersion and surface brightness.
- These relations must be calibrated locally using other distance indicators, Cepheids or surface brightness fluctuations; then they can be extended into the general Hubble flow regime.
- Their origins-and thus there universality -are not yet well understood. There may be some systematic variations.

The basic idea:

- Need a correlation between a distance-independent quantity "X", for example, temperature or color for stars in the H-R diagram, or the period for the Cepheids, and a distance-dependent one. Why for example, stellar absolute magnitude, M.

- Two sets of objects at different distances will have a systemic shift in the apparent versions of why that is, stellar apparent magnitude, am from which we can deduce the relative distance.

- This works for stars, main sequence fittings,.-Luminosity relations, we can we find such relationships for galaxies?



The Tulley-Fisher Relationship - Galaxy Distance vs Kinematic Rotational Speed

A new method of determining distances to galaxies, Tulley, Fisher, Astron and AstroPhys, Vol.54, p.661-673, 1977

Tulley -Fisher is a correlation that holds for galaxies with disks (spiral galaxies) stabilized by rotation, between the **intrinsic luminosity L of the galaxy** in optical or near-infrared bands and the **rate of rotation W**.

- A well-defined **Luminosity versus Rotational Speed** often measured as H1 21 cm line with **relation for spirals**:

$$L \approx v_{rot}^{\gamma}, \quad \gamma \approx 4 \text{ varies with wavelength.}$$

Or: $$M = b \log(W) + c, \text{ where:}$$

- **M** is the absolute magnitude
- **W** is the **Doppler broadened line** with, typically measured using the HI 21 cm line, corrected for inclination,

$$W_{true} = W_{obs}/\sin(i)$$

- Both the **slope b** in the **zero-point c** can be measured from the set of nearby spiral galaxies with well-known distances.
- The slope b can also be measured from any set of galaxies with roughly the same distance-for example, galaxies in the cluster-even if that distance is not known.

- *Scatter* is approximately 10 to 20% at best, which limits the accuracy.
- Problems include dust extinction, so working in the redder bands is better.

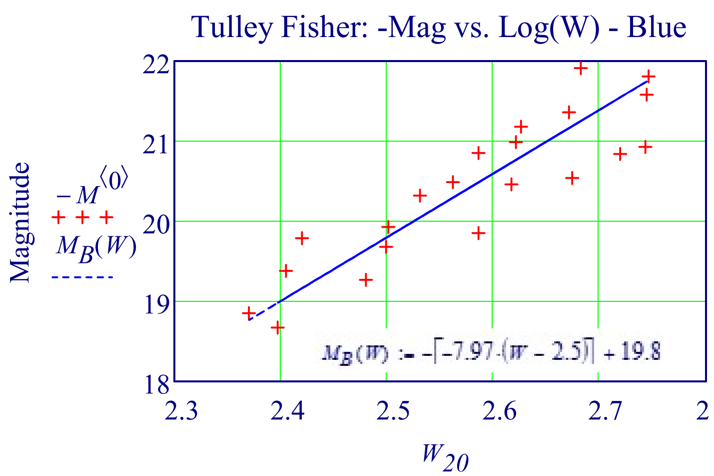
XXIV. The Calibration of Tully-Fisher Relations and The Value of The Hubble Constant

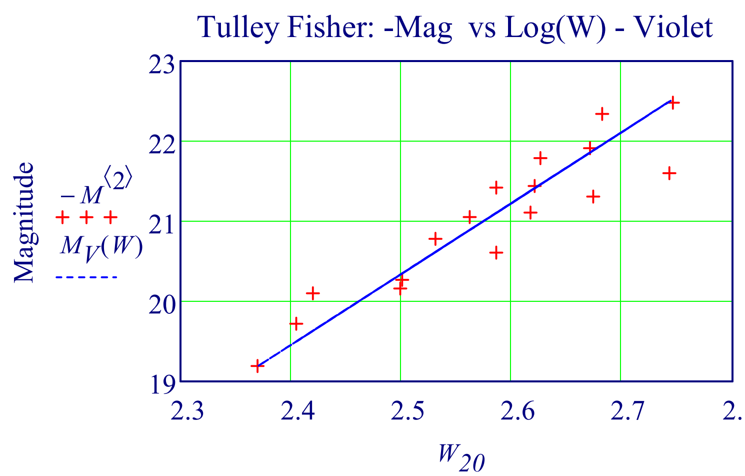
THE ASTROPHYSICAL JOURNAL, 529:698È722, 2000 February 1

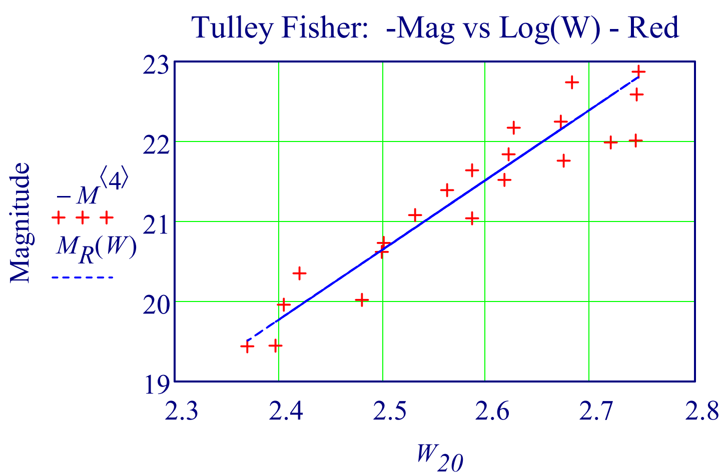
Photometric and Kinematical Data for Tulley Fisher Calibrators for Different Color Bands of 21 cm H I Line

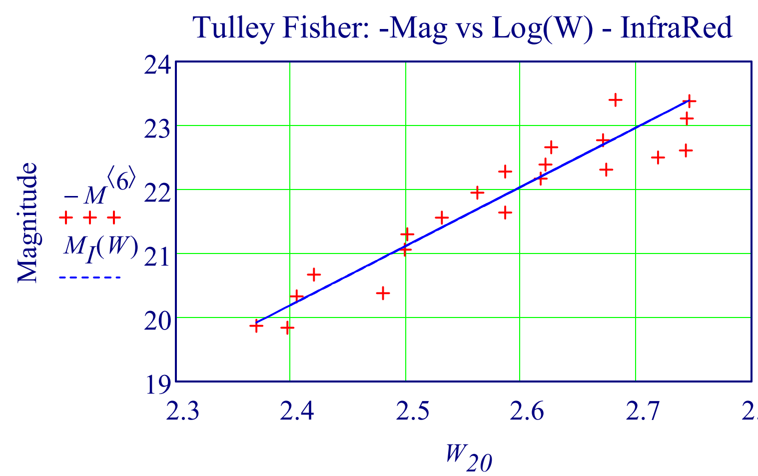
B_T^c (mag)	V_T^c (mag)	R_T^c (mag)	I_T^c (mag)	$H_{-0.5}^c$ (mag)	$\log W_{20}^c$ (km s^{-1})	$\log W_{50}^c$ (km s^{-1})	$i_I{}^a$ (deg)	$\bar{i}^a$ (deg)

$M := READPRN(\text{"Tulley Fisher Data A-Org4.txt"})$ $\quad W_{20} := M^{\langle 10 \rangle}$ $\quad W := W_{20}$ $\quad cols(M) = 18$





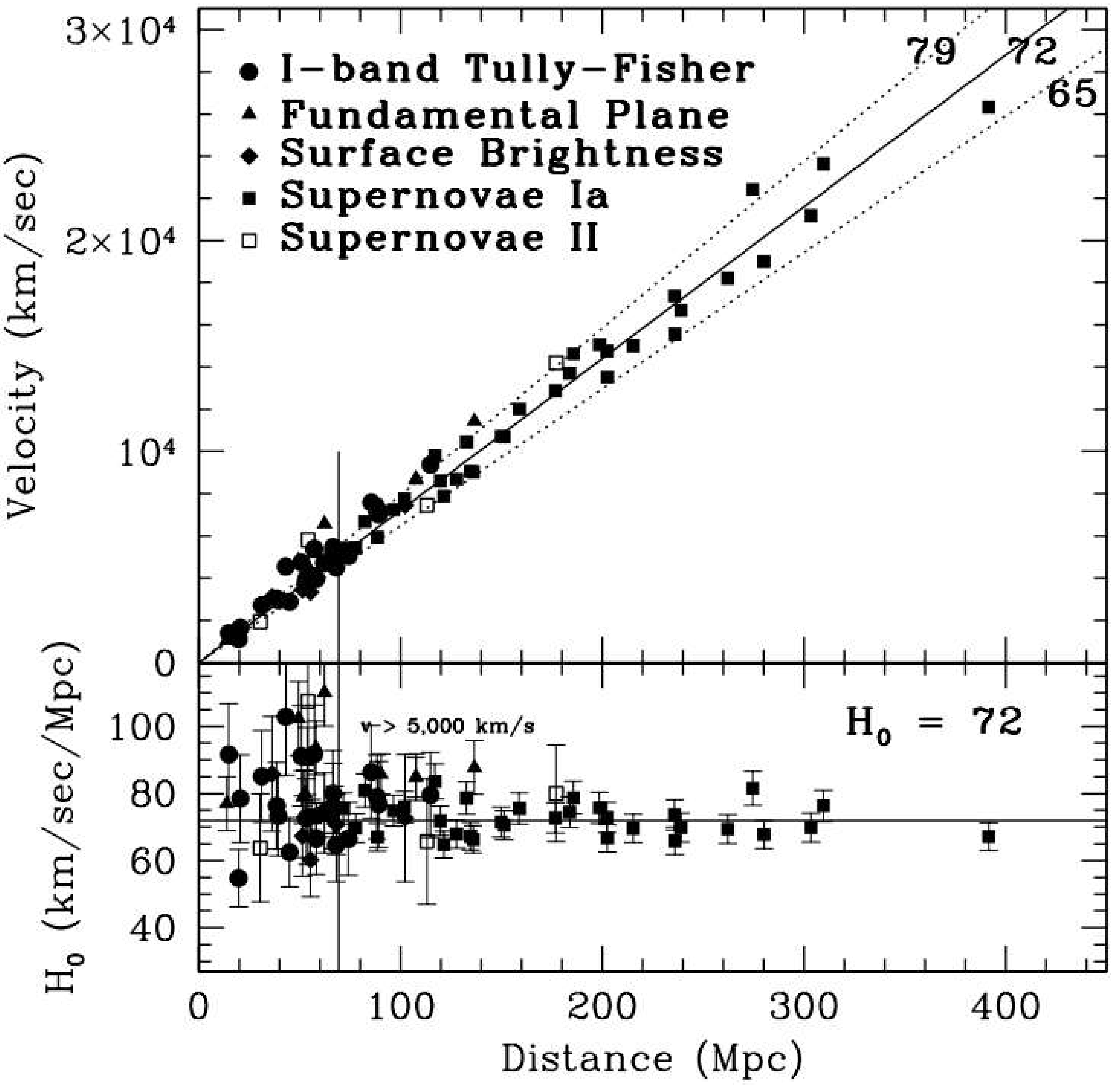




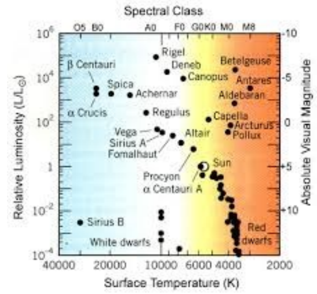
The Tully-Fisher Relation and Its Historical Importance

The Tully-Fisher empirical correlation between the **luminosity and rotational velocity of spiral galaxies** serves as a **distance indicator** to measure distances **independent of redshift**. **Possible Mechanism:** Rotational Velocity related to Mass, which is related to Luminosity. The Tully-Fisher relation has played an important role in Hubble constant measurements since its inception. In 1977 Brent Tully and Richard Fisher published their paper, They used only inclined spiral galaxies and proposed the usage of the linear relation between H I (21-cm neutral hydrogen (H I) emission line) profile and absolute magnitude as a distance indicator. The publication of the Tully-Fisher relation and the proposal to use it as a distance indicator was significant in many different ways. Firstly, it provided a robust new tool for measuring distance at redshifts that other methods such as Cepheid variable stars cannot. Secondly, Tully and Fisher measured the Hubble constant H_0 to be 80 km $s^{-1}Mpc^{-1}$ from the Virgo cluster and Ursa Major. This value was the first to deviate from the two mainstream values. It was also used to probe the distribution and properties of dark matter in galaxies.

I-band Tully-Fisher



Stellar Mass, Luminosity, and Lifespan



The H-R diagram is a plot of the luminosity up and down, versus temperature left and right with increasing temperature going to right to the left, and cooler stars on the right, hotter stars on the left, dimmer stars at the bottom and, and brighter stars (more luminous) at the top. Importnat: The main sequence is composes up to 80% of all the stars. Giants, supergiants, and white dwarfs are a minority.

Data Source: Accurate masses and radii of normal stars: Modern results and applications, G. Torres

"We have identified 95 detached binary systems containing 190 stars (94 eclipsing systems, and α Centauri) that satisfy our criterion that the mass and radius of both stars be known to an accuracy of ± 3% or better."

Binary Star	P(d)	Mass	±	Radius	±	Teff	±	log g	±	log L	±	MV	±
Data Table:	Vmax	(M⊙)	(M⊙)	(R⊙)	(R⊙)	(K)	(K)	(cgs)	(cgs)	(L⊙)	(L⊙)	(mag)	(mag)

Vmax is the Apparent Visual Magnitude

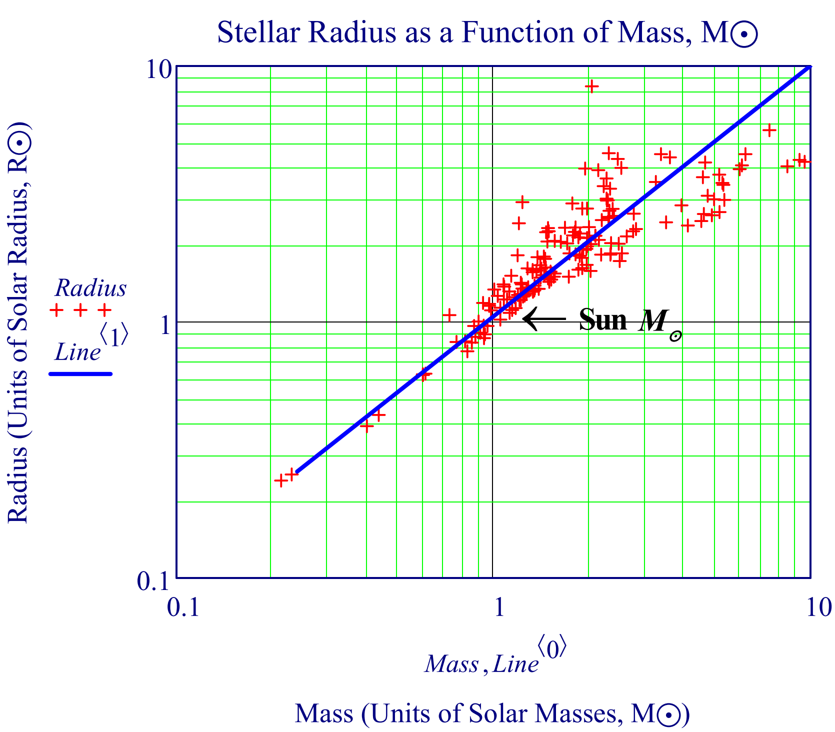
$L_\odot := 3.846 \cdot 10^{26} W$

Read in Data from File: $ObsParam := READPRN(\text{"Observed Parameters - Final.txt"})$

$Mass := ObsParam^{\langle 1 \rangle}$ $Radius := ObsParam^{\langle 3 \rangle}$ $Teff := ObsParam^{\langle 5 \rangle}$ $logL := ObsParam^{\langle 9 \rangle}$ $M_v := ObsParam^{\langle 11 \rangle}$

Dependence of stellar radius on mass for **Main-Sequence** stars. Actual measurements show that the radius increases nearly in proportion to the mass over much of the range (as indicated by the straight line drawn through the data points). Most stars are members of binary systems—where two stars orbit one another, bound together by gravity. Here we describe—in an idealized case where the relevant orbital parameters are known—how we can use the observed orbital data, together with our knowledge of basic physics, to determine the masses of the component stars

Plot Stellar Data for Binary Stars from Above Torres Paper



Binary Star Mass Relationship:

Given Mass M, Period P, and semimajor axis A, then Kepler's Law can be used to deduce relationships about binary star masses:

$$M_1 + M_2 \approx \frac{a^3}{p^2}$$

Example Sirius Binaries A & B

- orbital period = 50 years
- seimi-major axis = 20 AU
- $M_a + M_b = 3.2\, M_\odot$
- further study reveals: $M_a = 2.1\, M_\odot$ and $M_b = 1.1\, M_\odot$

For Main Sequence Stars,

the Stellar plot shows that relative to the Sun, with a mass of 1 $M_\odot$ and a size of 1 $R_\odot$, the mass and radius of Main Sequence Stars is only 0.1X to 10X relative to the sun.

Stellar Masses of Main Sequence

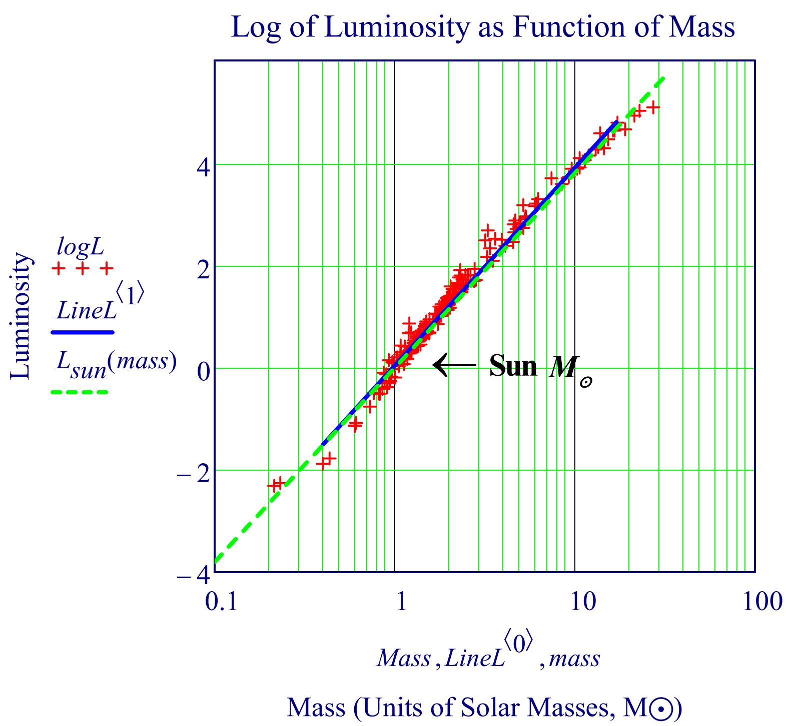
The following **Luminosity vs. Mass** plot shows a huge variation of 7 orders of magnitude of Luminiosity versus about 1.5 orders of magnitude of change for the Mass. This also suggests there must be a large variation with temperature verus mass. Mass is the Main Determinate of where a star will lie on the Main Sequence.

But for Luminosity:

$$L = 4\pi \cdot R^2 \cdot \sigma \cdot T^4$$

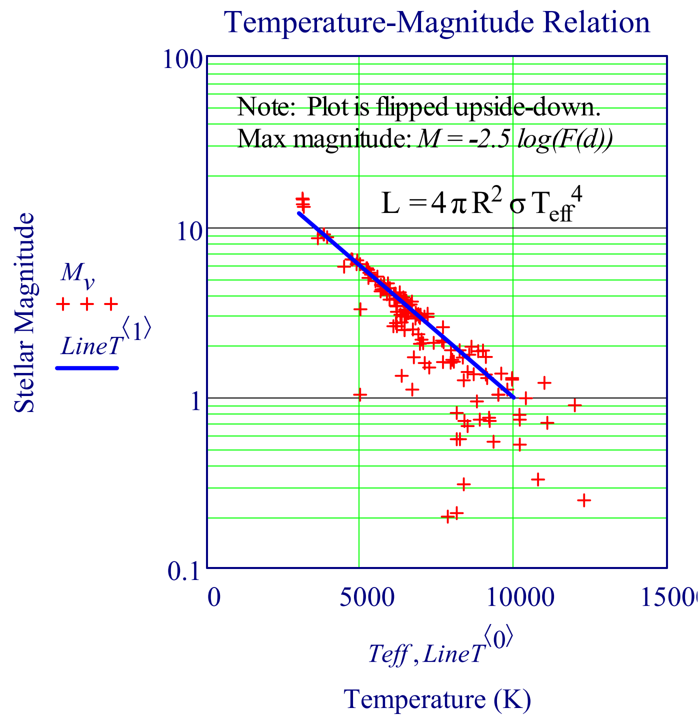
$$\frac{L}{L_\odot} \approx \left(\frac{M}{M_\odot}\right)^4 \qquad L_{sun}(M_s) := \log\left(M_s^{3.8}\right)$$

Plot Binary Stellar L vs Mass Data from Above Torres Paper

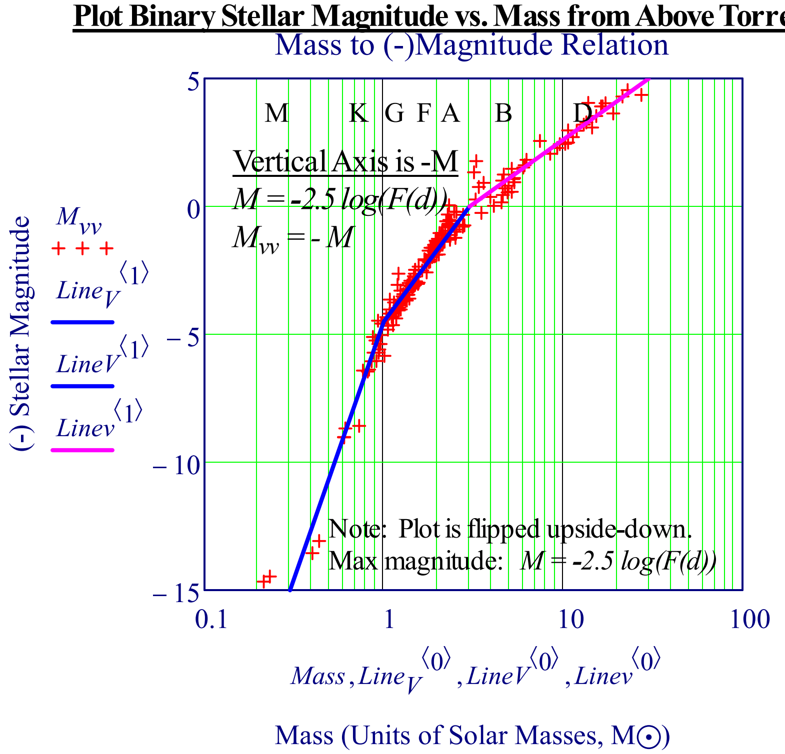


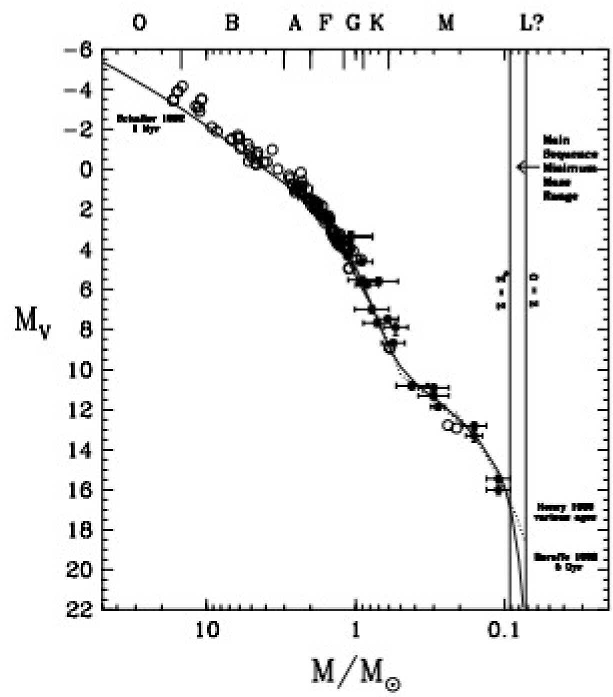
The Mass-Luminosity Relation from End to End

The Components of Close Binary Stars
ASP Conference Series, Vol. 318, 2004, Todd Henry



Plot Binary Stellar Magnitude vs. Mass from Above Torres Paper





Stellar Lifetimes

Luminosity increases as Mass^3 for massive main-sequence stars and Mass^4 for more common main-sequence stars
Total fuel to burn in star is the mass. Therefore:

A 5 solar mass, $M_\odot$, star has only five times more hydrogen fuel than the Sun,

but (the star's luminosity)/(the Sun's luminosity) $= (5/1)^4 = 625$!

Its lifetime $= 1/(5/1)^{(4-1)} \times 10^{10}$ years $= (1/125) \times 10^{10}$ years $= 8.0 \times 10^7$ years.

More massive stars burn up fastest and have shortest lives since the luminosity increases as the cube of the mass for the most massive stars.

The Luminosity Density

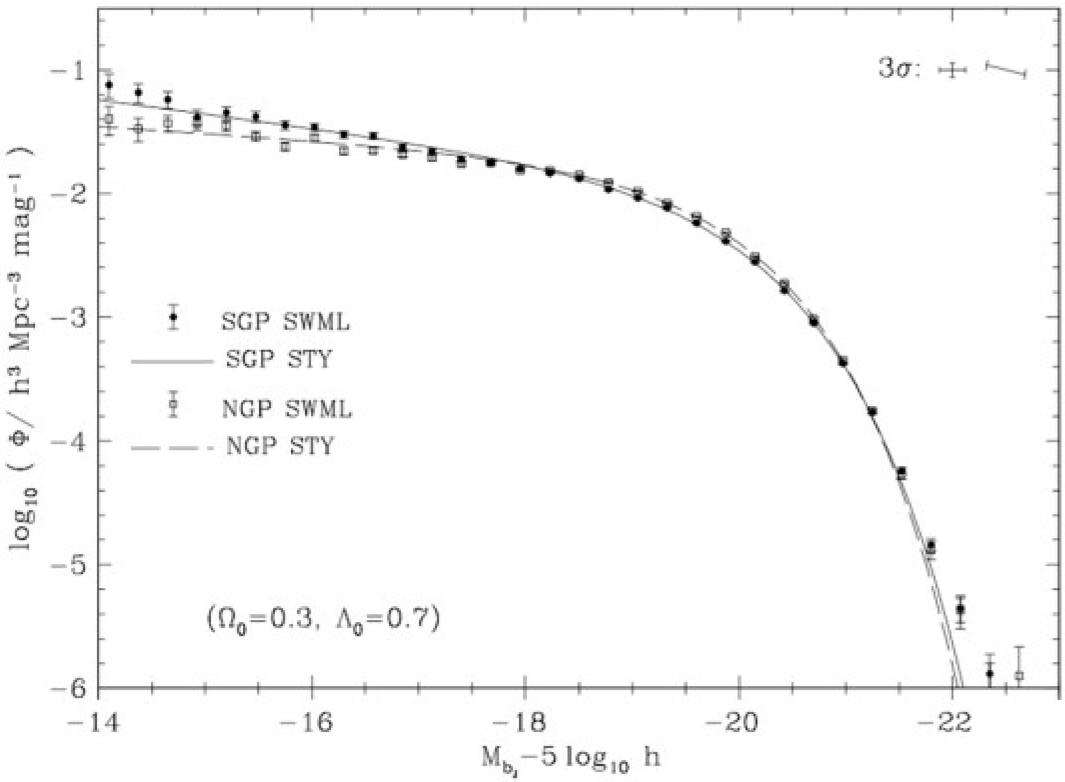
Ay21 Lecture 7: The Contents of the Universe

https://sites.astro.caltech.edu/~george/ay21/Ay21_Lec07.pdf

The Contents of the Universe Evolve

The relative abundances of different components change in time, due to their different EOS behavior:

Integrate galaxy luminosity function (obtained from large redshift surveys) to obtain the mean luminosity density at $z \approx 0$



SDSS, r band: $\rho_L = (1.8 \pm 0.2) \times 10^8\, h_{70}\, L_\odot/\mathrm{Mpc}^3$

2dFGRS, b band: $\rho_L = (1.4 \pm 0.2) \times 10^8\, h_{70}\, L_\odot/\mathrm{Mpc}^3$

The Local Mass Density of the Luminous Matter in Galaxies: $\Omega_{0,lum}$

$$\rho_{lum} = \rho_L \times \langle M/L \rangle \times \langle 1 + f_{gas} \rangle \approx (7 \pm 2) \times 10^8\, h_{70}\, M_\odot/\mathrm{Mpc}^3$$

$$\rho_{lum} \approx (4.7 \pm 1.3) \times 10^{-32}\, h_{70}\ \mathrm{g\ cm^{-3}}$$

Recall that $\rho_{0,crit} = 3H_0^2/(8\pi G) = 0.921 \times 10^{-29}\, h_{70}^2\ \mathrm{g\ cm^{-3}}$

$\Omega_{0,lum}$ Luminous Baryon Density $\approx$ **(0.0051 0.0015) h_{70}^{-1}**

All of the visible matter amounts to only half a percent

of the total mass/energy content of the universe!

(Interestingly, this may be about the same as the contribution from the massive cosmological neutrinos…)

Luminosity Function of Galaxies -The Schechter Luminosity Function

Definition of the luminosity function. The luminosity function specifies the way in which the members of a class of objects are distributed with respect to their luminosity. More precisely, the luminosity function is the number density of objects (here galaxies) of a specific luminosity. $\phi(M)\,dM$ is defined as the number density of galaxies with absolute magnitude in the interval $|M, M + dM|$. **The total density of galaxies is then**

$$\nu = \int_{-\infty}^{\infty} dM\ \Phi(M)$$

Accordingly, **$\phi(L)\,dL$ is defined as the number density of galaxies with a luminosity between L and L + dL**. It should be noted here explicitly that both definitions of the luminosity function are denoted by the same symbol, although they represent different mathematical functions, i.e., they describe different functional relations. It is therefore important (and in most cases not difficult) to deduce from the context which of these two functions is being referred to.

Problems in determining the luminosity function of Galaxies

At first sight, the task of determining the luminosity function of galaxies does not seem very difficult. The history of this topic shows, however, that we encounter a number of problems in practice. As a first step, the determination of galaxy luminosities is required, for which, besides measuring the flux, distance estimates are also necessary. For very distant galaxies redshift is a sufficiently reliable measure of distance, whereas for nearby galaxies the methods discussed earlier have to be applied. Another problem occurs for nearby galaxies, namely the large-scale structure of the galaxy distribution. To obtain a representative sample of galaxies, a sufficiently large volume has to be surveyed because the galaxy distribution is heavily structured on scales of 100^{h-1} Mpc and more. On the other hand, galaxies of particularly low luminosity can only be observed locally, so the determination of $\phi(L)$ for small L always needs to refer to local galaxies. Finally, one has to deal with the so-called *Malmquist bias*; in a flux- limited sample luminous galaxies will always be overrep- resented because they are visible at larger distances (and therefore are selected from a larger volume). A correction for this effect is always necessary, and was applied to the Figure below.

The Schechter Luminosity Function

An Analytic Expression For The Luminosity Function For Galaxies. Paul Schechter

The global galaxy distribution can be roughly approximated by the Schechter luminosity function →

$$\Phi(L) = \left(\frac{\Phi^*}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} \exp\left(-L/L^*\right)$$

where L^* is a characteristic luminosity above which the distribution decreases exponentially, α is the slope of the luminosity function for small L, and ϕ^*_s specifies the normalization of the distribution. A schematic plot of this function, as well as a fit to early data, is shown in the Figure below. Expressed in magnitudes, this function appears much more complicated. Considering that an interval dL in luminosity corresponds to an interval dM in absolute magnitude, with $dL/L = -0.4 \ln 10\ M$, and using $\phi(L)\,dL = \phi(M)\,dM$, i.e., the number of sources in these intervals are of course the same, we obtain

$h := 0.74$

$\phi_s := 1.6\cdot 10^{-2} h^3 \cdot Mpc^{-3}$ $\qquad \phi_s := 1000$

$M_{sB} := -19.7 + 5\log(h)$ $\qquad \alpha := -1.07$ $\qquad \phi(M) := 0.921\,\phi_s \cdot 10^{0.4\cdot(\alpha+1)(M_{sB}-M)} \cdot e^{\left[-10^{0.4(M_{sB}-M)}\right]}$

$M_{sB} = -20.354$

$L_{sB} := 1.2\cdot 10^{10} \cdot h^{-2} \cdot L_s$

Elliptical cD Type Galaxies

These are extremely luminous (up to MB -25) and large (up to R < 1 Mpc) galaxies that are only found near the centers of dense clusters of galaxies. Their surface brightness is very high close to the center, they have an extended diffuse envelope, and they have a very high M/L ratio. It is not clear whether the extended envelope actually 'belongs' to the galaxy or is part of the galaxy cluster in which the cD is embedded, since such clusters are now known to have a population of stars located outside of the cluster galaxies.