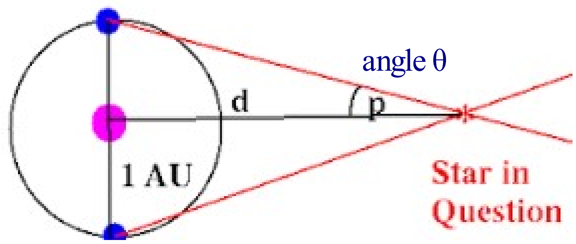
X. Measurement of Cosmic Distances - Trigometric Parallax

The most important fundamental distance measurements come from trigonometric parallax. As the Earth orbits the Sun, the position of nearby stars will appear to shift slightly against the more distant background. When a star is observed from two points separated by a distance ***b***, the star's apparent position will shift by an angle ***θ***. If the baseline of observation is perpendicular to the line of sight to the star, the parallax distance will be

$$lightyear := 9.46 \cdot 10^{12} km \qquad pc := 3.261 lightyear \qquad arcsec := \frac{\circ}{3600}$$

An astronomical unit (AU, or au), a unit of length effectively equal to the average, or mean, distance between the Earth and the sun.

$b := 2AU$ Earth's orbit (b = 2 AU) as a baseline, b



Parallax Distance, d_π

$$d_\pi(\theta, b) := 3.261 lightyear \cdot \left(\frac{b}{1AU}\right) \cdot \left(\frac{\theta \cdot \circ}{1 arcsec}\right)^{-1}$$

Measuring the distances to stars (galaxies are far too distant to be located by parallax) using the Earth's orbit ($b = 2\,AU$) as a baseline is a standard technique. Since the size of the Earth's orbit is known with great accuracy from radar measurements, the accuracy with which the parallax distance can be determined is limited by the accuracy with which ***θ*** can be measured. The Hipparcos satellite, launched by the European Space Agency in 1989, found the parallax distance for ~ 10^5 stars, with an accuracy of ~ 1 milliarcsecond.

Two decades after the end of the Hipparchos mission, another breakthrough arrived. In 2013, ESA launched a telescope called Gaia that charts the positions, parallaxes, and proper motions of more than one billion stars. That number represents only about 1% of the actual number of stars in the galaxy, but that's enough for astronomers to extrapolate the observations to understand how the Milky Way behaves as a whole. Using Gaia data, they could, for the first time, create a dynamic movie of the galaxy's life over billions of years, uncovering past events but also projecting what will happen in the future.
"Hipparcos had a detector with only one pixel and could only observe one star at a time," said de Bruijne, who is ESA's deputy project scientist for the Gaia mission. "Gaia, on the other hand, has nearly a billion pixels in its detectors and can observe thousands of stars at the same time."
Gaia's mirrors are 20 times larger and therefore it collects light much more efficiently than its predecessor, seeing much deeper into the galaxy.

In terms of absolute maximum distances, **Very Long Baseline Interferometry (VLBI)** can push these limits even further, potentially up to around **30,000 light years** with current technologies if the parallax measurement precision can be maintained at about (10 thousands of an arc second). Uses radio wavelengths 90 cm to 3 mm.
See Section XIX B. Our Galactic Home - The Milky Way

The **Event Horizon Telescope (EHT)** is a large telescope array consisting of a global network of radio telescopes. The EHT project combines data from several very-long-baseline interferometry (VLBI) stations around Earth, which form a combined array with an angular resolution sufficient to observe objects the size of a supermassive black hole's event horizon. The project's observational targets include the two black holes with the largest angular diameter as observed from Earth: the black hole at the center of the supergiant elliptical galaxy Messier 87 (M87*, pronounced "M87-Star").

Parallax Distance: $d_\pi\left(10 \cdot 10^{-3} arcsec, b\right) = 37368 \cdot lightyear$

An Improved Distance to NGC 4258, The Astrophysical Journal Letters, 2019 December 1
This paper claims a distance estimate of 7.6 Mpc or **24 Million Lightyears** for NGC 4258.

Measurement of Cosmic Distances - The Standard Candle

MEASURING COSMOLOGICAL PARAMETERS

The current proper distance to a galaxy, $d_p(t_0)$, is not a measurable property

Since cosmology is ultimately based on observations, if we want to find the distance to a galaxy, we need some way of computing a distance **from that galaxy's observed properties**. Let's focus on the properties that we can measure for objects at cosmological distances. We can measure the flux of light, *f*, from the object, in units of watts per square meter. The complete flux, **integrated over all wavelengths** of light, is called the **bolometric flux**. (A bolometer is an extremely sensitive thermometer capable of detecting electromagnetic radiation over a wide range of wavelengths.)

Cosmologists would like to know the scale factor *a(t)* for the universe. For a model universe whose contents are known with precision, the scale factor can be computed from the Friedmann equation. Finding *a(t)* for the real universe, however, is much more difficult. **The scale factor is not directly observable**; it can **only be deduced** indirectly from the **imperfect and incomplete observations** that we make of the universe around us.

The Standard Candle

One way of using measured properties to assign a distance is the standard candle method. A standard candle is an object whose luminosity *L* is known. For instance, if some class of astronomical object had luminosities which were the same throughout all of space-time, they would act as excellent standard candles – if their unique luminosity *L* were known. For instance, if some class of astronomical object had luminosities which were the same throughout all of space-time, they would act as excellent standard candles – if their unique luminosity *L* were known. Nowadays, the bolometric apparent magnitude of a light source is defined in terms of the source's bolometric flux, *m*,

$$m \equiv -2.5 \log_{10}(f/f_x)$$

Reference Flux: $f_x := 2.53 \cdot 10^{-8} \cdot \frac{W}{m^2}$

where the reference flux f_x is set at the value $f_x = 2.53 \times 10^{-8}$ *watt* m^{-2}. Thanks to the **negative sign** in the definition, a small value of m corresponds to a large flux *f*. For instance, the flux of sunlight at the Earth's location is f = 1367 watts m^{-2} ; the Sun thus has a bolometric apparent magnitude of m = −26.8.
The bolometric absolute magnitude of a light source is defined as the apparent magnitude that it would have if it were at a **luminosity distance** of d_L = 10 pc. Thus, a light source with luminosity *L* has a bolometric absolute magnitude, *M*. Luminosity of the sun: $L_{M\odot}$

Reference Luminosity: $L_\odot := 3.846 \cdot 10^{26} W$ $\quad L_x := 78.7 \cdot L_\odot$ $\quad M \equiv -2.5 \log_{10}(L/L_x)$

Since that is the luminosity of an object which produces a flux $f_x = 2.53 \times 10^{-8}$ *watt* m^{-2} when viewed from a distance of 10 parsecs. The bolometric absolute magnitude of the Sun is thus M = 4.74.

Given the definitions of apparent and absolute magnitude, the relation between an object's **apparent magnitude, *m*,** and its absolute magnitude, *M*, can be written in the form

$$M = m - 5\log\left(\frac{d_l}{10pc}\right)$$

The Distance Modulus is Defined as *m − M*, and is related to the luminosity distance by the relation

$$m - M = 5\log\left(\frac{d_l}{10pc}\right) + 25$$

Using standard candles to determine the Hubble constant is the method used by Hubble himself.
The recipe for finding the Hubble constant is a simple one:

- Identify a population of standard candles with luminosity *L*.
- Measure the redshift **z** and flux ***f*** for each standard candle.
- Compute $dL = (L/4\pi f)^{1/2}$ for each standard candle.
- Plot c_z versus *dL* .
- Measure the slope of the c_z versus *dL* relation when $z << 1$; the slope gives H_0 .

Initial Mass Function, IMF

The properties and evolution of a star are closely related to its mass.

In astronomy, the initial mass function (IMF) is an **empirical function** that describes the **initial distribution of masses for a population of stars** during star formation. IMF not only describes the formation and evolution of individual stars, it also serves as an important link that describes the **formation and evolution of galaxies**. The mass of a star can **only be directly determined** by applying **Kepler's third law into binary stars system**. However, the number of binary systems that can be observed is low, thus **not enough samples to estimate** the initial mass function. Therefore, **stellar luminosity function is used to derive a mass function** (present-day mass function, PDMF) by applying mass–luminosity relation. the luminosity function requires accurate determination of distances, and the most straightforward way is by measuring stellar parallax within 20 parsecs from the earth. The IMF is often stated in terms of a **series of power laws**, where **$\xi(m)\Delta m$, the number of stars with masses in the range m to $m + dm$** within a **specified volume** of space, is proportional to $m^{-\alpha}$.

$$\xi(\log m) = \frac{d(N/V)}{d \log m} = \frac{dn}{d \log m}$$

Note: The vertical axis for the Initial Mass Function $\xi(m)$ is **SCALED** so that **for m greater than** $M_\odot$ it is $(m/M_\odot)^{-2.35}$

Edwin E. Salpeter (1955) was the first astrophysicist who attempted to **quantify IMF by applying power law** into his equations. ξ_0 is a constant relating to the local stellar density

$$M_\odot := 1.989 \cdot 10^{30} kg \qquad \xi_0 := 1 \qquad \xi(m, \Delta m) := \xi_0 \cdot \left(\frac{m}{M_\odot}\right)^{-2.35} \cdot \left(\frac{\Delta m}{M_\odot}\right) \qquad \xi_S(m) := \xi_0 \cdot \left(\frac{m}{1}\right)^{-2.35}$$

Kroupa (2001)

$$\xi_K(m) := if\left[m < 0.08, m^{-0.3} \cdot 15, if\left[(m \ge 0.08) \wedge (m \le 0.5), 1.3 \cdot m^{-1.3}, m^{-2.35}\right]\right]$$

***Intro to Cosmology,* 2nd. Ed., Ryden 2016 Equation 7.3**

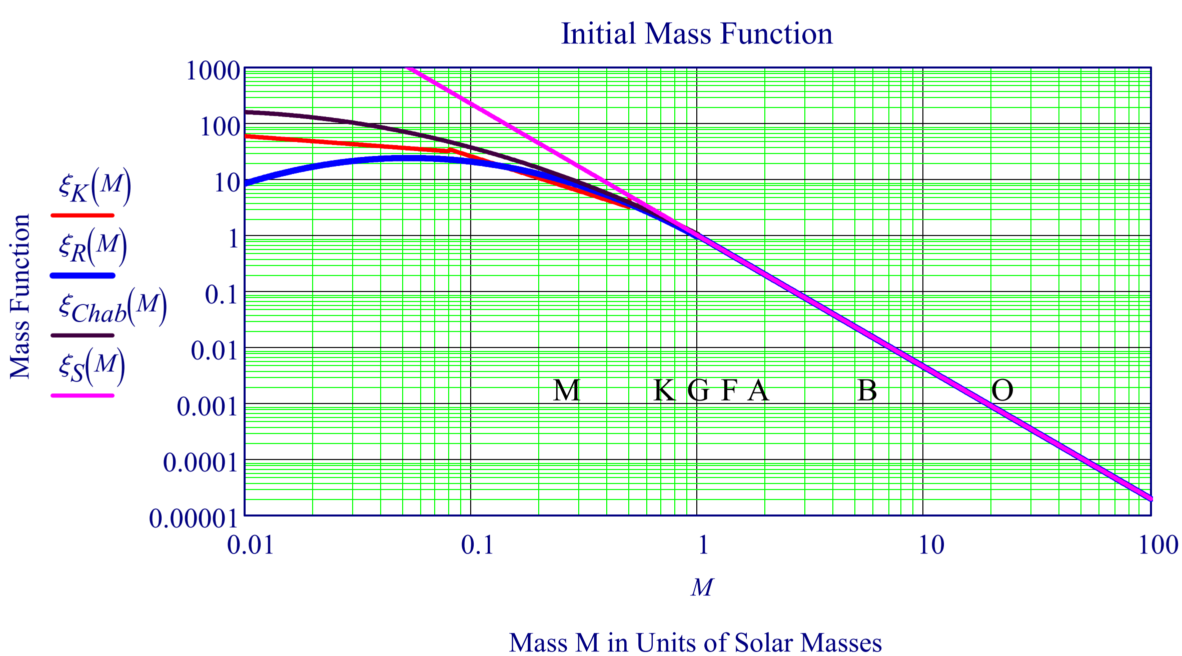
$$\xi_r(M) := 2.5 \cdot \frac{1}{M} \cdot exp\left[\frac{-(log(M) - log(0.2))^2}{2 \times 0.5^2}\right] \qquad \xi_R(M) := if\left(M \ge 1, M^{-2.35}, \xi_r(M)\right)$$

Chabrier (2003) gave the following expression for the density of individual stars in the Galactic disk, in units of parsec^{-3}

The vertical axis is not ε(m), but is a scaled version $(m/M_\odot)^{-2.35}$

$$\xi_{chab}(M) := 55 \cdot \frac{0.158}{M \cdot ln0(10)} \cdot exp\left[\frac{-(log(M) - log(0.08))^2}{2 \times 0.69^2}\right] \qquad \xi_{Chab}(M) := if\left(M \le 1, \xi_{chab}(M), M^{-2.35}\right)$$

Mass ranges corresponding to the standard stellar spectral types O through M are indicated.



Initial Mass Function

Massive O stars are **extremely luminous**, they are also **short-lived**. An O star with a mass $M = 60 M_\odot$ will run out of fuel for fusion in a time $t \approx 3$ Myr; it will then explode as a type II supernova.

Note: $\Omega_{stars} \approx 0.3\%$