XI. Cosmic Distance Scale - Standard Candle 1: Cepheid Variables

The Standard Candle

To move outward in distance one starts One, with trigonometric parallaxes, then observes the same object with the other types of less precise parallaxes to calibrate and scale them. Once this is done one has the distance ladder reaching about 10,000 pc – **halfway across the Milky Way**. At this point one must put aside the parallax method and use other methods. With few exceptions, distances based on direct measurements are available only out to about a thousand pc, which is a modest portion of our own Galaxy. For distances beyond that, measurements are going to **depend upon physical assumptions**, that is, **knowledge of the object** in question. One must recognize the object and assume the **class of objects is homogeneous enough** that its members can be used for a meaningful estimation of distance – a standard candle as it were.

Almost all of the remaining rungs on the ladder are standard candles of one kind or another. A standard candle is an object that belongs to some class that has a **known brightness** (i.e., all members of the class have the same brightness). By comparing the **known luminosity of the latter to its observed brightness**, the distance to the object can be computed using the inverse square law.
Two problems exist for any class of standard candle. **The principal one is calibration**, determining exactly what the absolute magnitude of the candle is. This includes defining the class well enough that members can be recognized, and finding enough members with well-known distances that their true absolute magnitude can be determined with enough accuracy. The **second lies in recognizing members of the class**, and not mistakenly using the standard candle calibration upon an object which does not belong to the class. At extreme distances, which are where one most wishes to use a distance indicator, **this recognition problem can be quite serious.**

Standard Candle #1: Cepheid Variables

Cepheids were first noticed in 1784 in the constellation Cepheus in the northern sky, so these stars became known as "Cepheid variables." Cepheids are stars that periodically dim and brighten. In 1908 Henrietta Leavitt noticed a relationship between the brightness (or "luminosity") of a Cepheid variable star and its period for its pulsations in luminosity. They have a **unique waveform** and we can measure their period independent of how far away they are.
In the 1950s, astronomer Walter Baade discovered that the **nearby Cepheid variables used to calibrate the standard candle were of a different type than the more distant ones used to measure distances to nearby galaxies.** The nearby Cepheid variables were young, massive stars with much higher metal content than the distant old, faint ones. As a result, **the old stars were actually much brighter than believed,** and this had the ultimate effect of **doubling the distances** to the globular clusters, the nearby galaxies, and the diameter of the Milky Way.
Cepheids are luminous variable stars that **radially pulsate**. The strong direct relationship between a Cepheid's luminosity and its pulsation period makes them an important standard candle for Galactic and extragalactic sources. **Type I Cepheids** undergo pulsations with **very regular periods** on the order of days to months.
A relationship between the **period and luminosity for Type I Cepheids** was discovered in 1908 by **Henrietta Swan Leavitt** in her investigation of thousands of variable stars in the Magellanic Clouds. To use them as standard candles, one **observes the pulsation period to get the luminosity (absolute magnitude)**. By then measuring the apparent brightness (value observed at Earth) one has everything needed to use the **distance modulus** $m - M$. The work was so important that Leavitt was considered for the Nobel Prize, but she died before her name could be submitted.

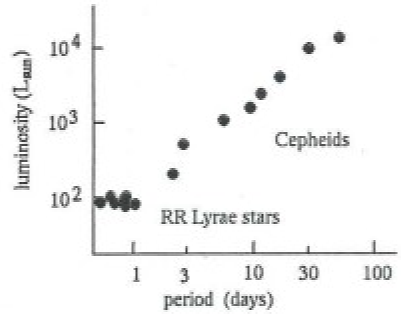
In addition, using data from the **HIPPARCOS astrometry satellite**, astronomers calculated the distances to many Galactic Cepheids using the trigonometric parallax technique. The resultant period-luminosity relationship for Type 1 Cepheids was:

$$M_V = 2.81 \log(P) - (1.43 \pm 0.1)$$

where M_V is the absolute magnitude and P is the period in days.

XII. Modeling the Dynamics of a Cepheid Variable

There are **two classifications** of variable stars, RR Lyrae and Oepheid Variables. BB Lyrae have approximately a Solar mass and are yellow-white giants with luminosities on the order of 100 times that of the Sun. **Cepheid Variables** are **yellow supergiants** with several Solar masses and luminosities on the order of 20,000 times that of the Sun. These stars pulsate as the result of a special relationship between **pressure and gravit**y. One idea is that as radiation emanates from the star, some of the He^{+} ionized into He^{2+} leading the surface of the star become more opaque.
As the surface darkens, less energy is able to escape therefore heating the gas within the star. As the gas heats it pushes outward expanding the staris radius. As the star grows in volume, the gas cools allowing the pressure inside to drop (He^{+2} converts back to He^{+}) and gravity to once again dominate by pulling everything inward. The cycle then is able to begin again.



Find The Period of a Cepheid Variable Star

From Newton's Second Law:

$$m\cdot\frac{d^2}{d\tau^2}R = \frac{-G\cdot M\cdot m}{R^2} + 4\pi R^2\cdot P$$

In Equilibrium R is constant

$$\frac{G\cdot M\cdot m}{R^2} = 4\pi R^2\cdot P$$

Let $R = R_0 + \delta R$ $\qquad P = P_0 + \delta P$

$$4\pi R_0\cdot P = \frac{G\cdot M\cdot m}{R_0^{\,3}}$$

$$m\cdot\frac{d^2}{d\tau^2}\left(R_0 + \delta R\right) = \frac{-G\cdot M\cdot m}{\left(R_0 + \delta R\right)^2} + 4\pi\left(R_0 + \delta R\right)^2\cdot\left(P_0 + \delta P\right)$$

First Order Approximation:
(Taylor Series Expansion)

$$\frac{1}{\left(R_0 + \delta R\right)^2} = \frac{1}{R_0^{\,2}}\cdot\left(1 - 2\cdot\frac{\delta R}{R_0}\right)$$

$$m\cdot\frac{d^2}{d\tau^2}(\delta R) = \frac{-G\cdot M\cdot m}{R_0^{\,2}} + \frac{2G\cdot M\cdot m}{R_0^{\,3}} + 4\pi R_0^{\,2}\cdot P_0 + 8\pi R_0\cdot P_0\cdot\delta R + 4\pi R_0^{\,2}\cdot\delta P$$

Substitute $\frac{G\cdot M\cdot m}{R^2} = 4\pi R^2\cdot P$

$$m\cdot\frac{d^2}{d\tau^2}(\delta R) = \frac{2G\cdot M\cdot m}{R_0^{\,3}} + 8\pi R_0\cdot P_0\cdot\delta R + 4\pi R_0^{\,2}\cdot\delta P$$

For the adiabatic expansion of a gas: $\quad P_0\cdot V_0^{\,\gamma} = P\cdot V^{\gamma} \qquad P\cdot V^{\gamma} = Constant \qquad V = \frac{4}{3}\cdot\pi\cdot R^3$

This Equation has the form of an Wave/Oscillation

$$P\cdot R^{3\gamma} = Constant \qquad \frac{\delta P}{P_0} = -3\gamma\cdot\frac{\delta R}{R_0} \qquad \frac{d^2}{d\tau^2}(\delta R) = -(3\gamma - 4)\cdot\frac{G\cdot M}{R_0^{\,3}}\cdot\delta R \qquad M := 10^6$$

Find the Period for this simple harmonic oscillation:

The Period, T, is $\quad T_{Cepheid} = \frac{2\pi}{\omega}$

Mass, M, and Radius, R, of Sun

$$M_\odot := 1.989\cdot 10^{30}kg \qquad R_\odot := 6.96\cdot 10^{8}m \qquad \gamma := \frac{5}{3}$$

$$\delta R(\tau) = A\cdot\sin(\omega\cdot\tau) \qquad \omega^2 = (3\gamma - 4)\cdot\frac{G\cdot M}{R_0^{\,3}}$$

For a Cepheid 10X Mass & 30X Radius of Sun

$$T_{Cepheid} := \frac{2\pi}{\sqrt{(3\gamma - 4)\cdot\frac{G\cdot 10M_\odot}{\left(30R_\odot\right)^3}}}$$

$$T_{Cepheid} = 6.024\cdot day$$

Modeling the Dynamics of a Cepheid: Solve for δ Radii, Velocity, and Pressure

Newton's Second Law

Use the Greek letter *thau* τ to represent the symbol for time (t)

$$m\cdot\frac{d^2}{d\tau^2}R = \frac{-G\cdot M\cdot m}{R^2} + 4\pi R^2\cdot P \qquad P_0\cdot V_0^{\gamma} = P\cdot V^{\gamma} \qquad V = \frac{4}{3}\pi\cdot R^3 \qquad P = P_0\cdot\left(\frac{R_0}{R}\right)^{3\gamma} \qquad R = R_0\cdot r$$

$$3\cdot\gamma = 5 \qquad P = P_0\cdot\left(\frac{1}{r}\right)^5 \qquad m\cdot R_0\cdot\frac{d^2}{d\tau^2}r = \frac{-G\cdot M\cdot m}{R_0^2\cdot r^2} + 4\pi R_0^2\cdot r^2\cdot P \qquad \frac{d^2}{d\tau^2}r = \frac{-G\cdot M}{R_0^3\cdot r^2} + \frac{4\pi R_0^2\cdot r^2\cdot P}{m}$$

$$P_0 := 56\cdot kPa$$

$$\frac{d^2}{d\tau^2}r + \frac{G\cdot M}{R_0^3\cdot r^2} - \frac{4\pi R_0^2\cdot P}{m\cdot r^3} = 0 \qquad \alpha_0 := \frac{G\cdot 10M_{\odot}}{(30R_{\odot})^3} \qquad \beta_0 := \frac{4\pi(30\cdot R_{\odot})^2\cdot P_0}{10M_{\odot}}$$

$$\frac{d^2}{d\tau^2}r + \frac{\alpha}{r(\tau)^2} - \frac{\beta}{r(\tau)^3} = 0 \qquad \alpha_0 = 1.457\times 10^{-10}\,\frac{1}{s^2} \qquad \beta_0 = 1.542\times 10^{-8}\cdot\frac{km}{s^2}$$

$$\alpha := 1.4\cdot 10^{-10} \qquad \beta := 1.2\cdot 10^{-10}$$

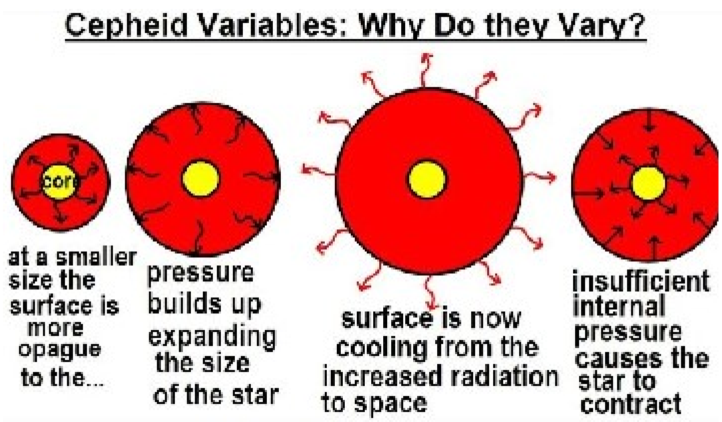
Solve Differential Equation for Cepheid Oscillations

Given $\quad r''(\tau) + \frac{\alpha}{r(\tau)^2} - \frac{\beta}{r(\tau)^3} = 0 \qquad r(0) = 1 \qquad r'(0) = 0$

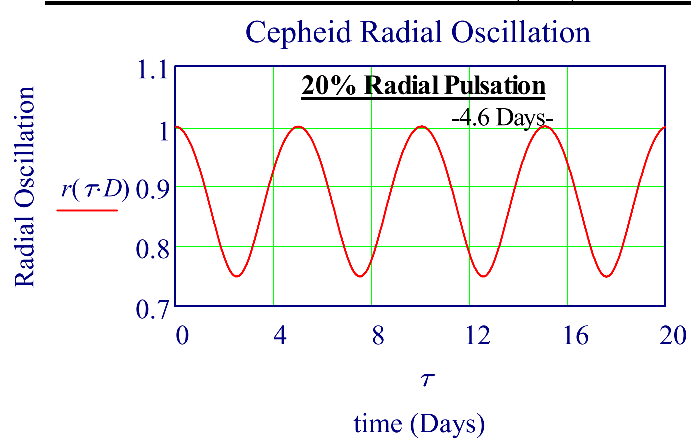
Ordinary Differential Equation Solver

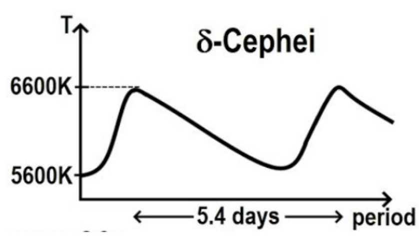
$$r := Odesolve(\tau, 10000000)$$

$$Day := 24\cdot 3600 \qquad D := Day$$



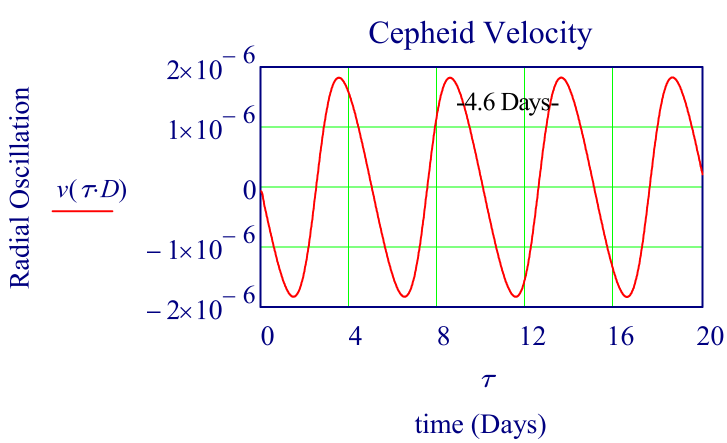
Below are Plots for Solution of Radius, Vel, Pressure

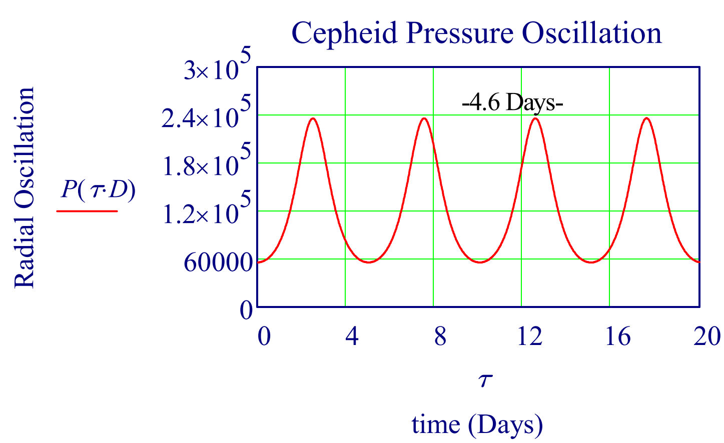




Definitions of Velocity, *v(τ)*, and Pressure, *P(τ)*

$$v(\tau) := \frac{d}{d\tau}r(\tau) \qquad P_{init} := 5.6\cdot 10^4 \qquad P(\tau) := \frac{P_{init}}{r(\tau)^5}$$





Calibrating Cepheid period-luminosity relation from the infrared surface brightness

Astronomy &Astrophysics 534, A95 (2011)   https://www.aanda.org/articles/aa/pdf/2011/10/aa17154-11.pdf

The Cepheid period-luminosity (P-L) Relation is fundamental to the calibration of the extra-galactic distance scale and thus to the determination of the Hubble constant.

DATA: Distances & absolute magnitudes Large Magellanic Clouds (LMC) Cepheids calculated using precepts

ID#	log(P)	d	σ(d)	$(m-M)_0$	σ(m−M)	M_V	M_I	M_J	M_H	M_K	W_{VI}	W_{JK}	E(B−V)	Δφ	Δ(m−M)
		(kpc)	(kpc)	(mag)	(mag)	(mag)	(mag)	(mag)	(mag)	(mag)	(mag)	(mag)	(mag)		(mag)

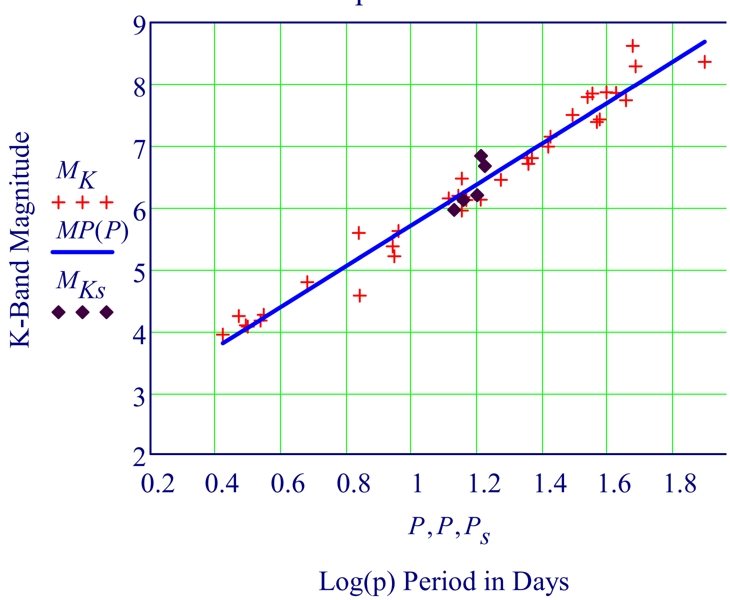
Read In Cepheid Data from File: $CPL := READPRN$("Distances and absolute magnitudes for the LMC Cepheids.txt"

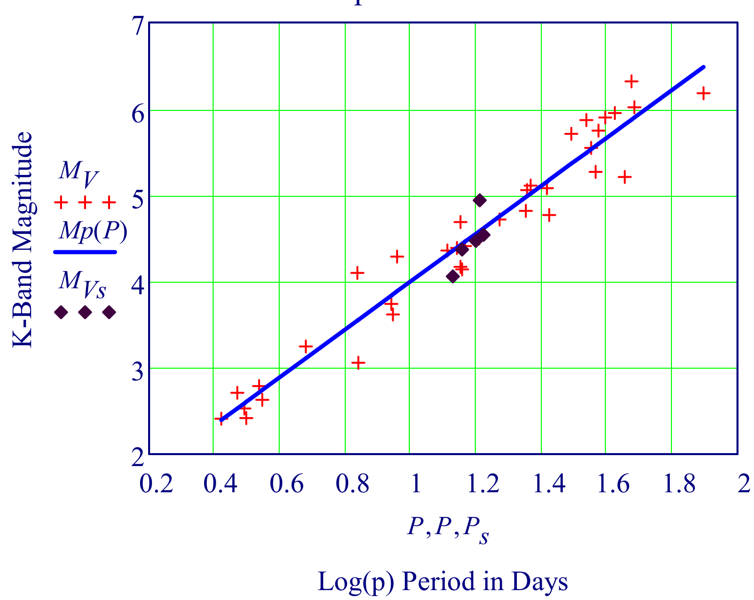
$P := CPL^{\langle 1 \rangle}$ $M_K := -CPL^{\langle 10 \rangle}$ $M_V := -CPL^{\langle 6 \rangle}$ $ab := line(P, M_K)$ $MP(p) := ab_1 \cdot p + ab_0$

$AB := line(P, M_V)$ $Mp(p) := AB_1 \cdot p + AB_0$ $ab_0 = 2.401$ $ab_1 = 3.315$ $AB_0 = 1.225$ $AB_1 = 2.774$

Read Small Magellanic Clouds: $CPL_s := READPRN$("Distances and absolute magnitudes for the SMC Cepheids.txt")

$P_s := CPL_s^{\langle 1 \rangle}$ $M_{Ks} := -CPL_s^{\langle 10 \rangle}$ $M_{Vs} := -CPL_s^{\langle 6 \rangle}$





Calibrating Cepheid period-luminosity relation, **Conclusion- J. Storm, W. Gieren, P. Fouqué:**

The emerging conclusion based on our data and analysis is that for accurate distance measurements to galaxies the **K-band Cepheid Period-Luminosity** is the best suited tool: it is metallicity-independent both regarding the slope and the zero point, it is very insensitive to reddening, and it has a smaller intrinsic dispersion than any optical PL relation.

Apparent Brightness

Describe how bright a star seems as seen from Earth by its apparent brightness. This is often called the **intensity** of the starlight. Sometimes it is called the **flux of light**.The apparent brightness is how much energy is coming from the star per square meter per second, as measured on Earth. The units are watts per square meter (W/m_2).

- the distance d to the star,
- the apparent brightness b of the star, and
- the luminosity L of the star.
- All of the energy produced by the star per second must cross a sphere of radius d.
- The study of geometry tells us that area of this sphere is $4\pi d^2$

$$b = \frac{L}{4\pi d^2}$$

$$L = \left(4\pi d^2\right) b$$